Permeability :-

Magnetic Circuit

The phenomenon of magnetism or electromagnetism are dependant upon certain property of a medium that is called permeability.

Absolute permeability (μ_0) = $4\pi \times 10^{-7}$

relative permeability (μ_r) = 1

Law of Magnetic force :-

The force between two magnetic poles is directly proportional to their pole strength, inversely proportional to the square of distance between them. and inversely proportional to the absolute permeability of the medium.

$$F \propto \frac{m_1 m_2}{\mu r^2}$$

$$\Rightarrow F = K \frac{m_1 m_2}{\mu r^2} \ N$$

$$\Rightarrow \hat{F} = K \frac{m_1 m_2}{\mu r^2} \hat{r}$$

$$\Rightarrow F = \frac{K m_1 m_2}{\mu r^2 \times r} \times r$$

$$\Rightarrow F = \frac{K m_1 m_2}{\mu r^3} \vec{r}$$

$$\Rightarrow F = \frac{K m_1 m_2}{\mu_0 \mu_r r^2} N$$

$$\Rightarrow \boxed{F = \frac{1}{4\pi} \frac{m_1 m_2}{\mu_0 \mu_r r^2} N}$$

Magnetic field strength (H) :-

Magnetic field strength at any point within a magnetic field is numerically equals to the force experienced by 'N' pole of 1 Wb placed at that point.

$$F = K \frac{1 \cdot M}{\mu r^2}$$

$$\Rightarrow \boxed{H = \frac{K M}{\mu r^2} \; N/Wb}$$

$\Rightarrow H = \frac{1}{4\pi} \frac{M}{\mu r^2}$ N/wb .

Magnetic potential :-

The magnetic potential at any point within a magnetic field is measured by the work done in sitting a pole of 1 wb from infinite to that point against the force of magnetic field .

$$M = \frac{Km}{\mu r}$$

$$\Rightarrow \boxed{M = \frac{m}{4\pi \mu r} \text{ J/wb}}$$

Flum per unit pole :-

suppose a unit pole radiate a flum of 1 wb then it is denoted as ϕ . If the pole radiate a flum of M wb .

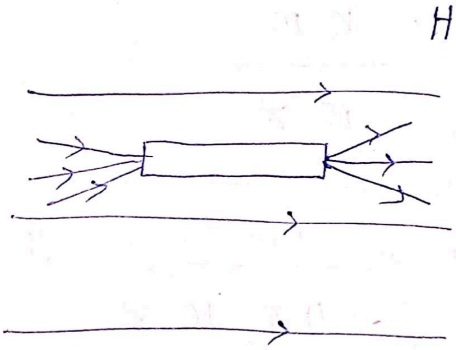
so ϕ = M wb .

Flux density :- (B)

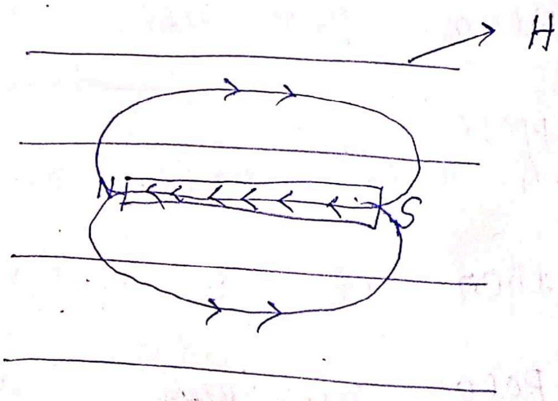
It is given that the flux passing per unit area through a plane right angle to it.

$$B = \frac{\phi}{A} \quad \frac{Wb}{m^2}$$

Absolute permeability (μ_0) and relative permeability (μ_r) :-



The above figure shows a bar magnetic material or a iron piece placed in uniform magnetic field strength of 'H' with flux density 'B'.



Well 'H' is present in the vacuum.

$$B_0 = \mu_0 H$$

Well the iron piece is magnetised by

induction, the magnetic flux density

$$B_i = \frac{m}{A}$$

so total,

$$B = B_0 + B_i$$

$$\Rightarrow B = \mu_0 H + \frac{m}{A}$$

$$B = \mu H$$

$$\Rightarrow B = \mu_0 \mu_r H$$

$$\Rightarrow B = (\mu_0 H) \mu_r$$

$$\Rightarrow B = B_0 \mu_r$$

$$\Rightarrow \mu_r = \frac{B \text{ (medium/ material)}}{B_0 \text{ (vacuum)}}$$

Intensity of Magnetisation (I) :-

It is defined as the induced pole strength developed per unit area.

m = pole strength induced in a magnet in weber.

A = pole area in mm^2

$$I = \frac{m}{A} \quad \frac{wb}{m^2}$$

Susceptibility (K) :-

Susceptibility is defined as the ratio of intensity of magnetisation (I) to magnetic force (H).

$$K = \frac{I}{H} \quad \frac{\text{Henry}}{\text{meter}}$$

Relation Between B, H, I, K :-

$$B = \mu H = \mu_0 H + \frac{m}{A}$$

$$\Rightarrow B = \mu_0 H + I$$

$$\mu = \frac{B}{H}$$

$$= \frac{\mu_0 H + I}{H}$$

$$= \mu_0 + \frac{I}{H}$$

$\Rightarrow \mu = \mu_0 + K$

$B = \mu H$

$= (\mu_0 + K) H$

$= \mu_0 H + K H$

$\Rightarrow B = B_0 + KH$

$B = \mu H$

$\mu = \mu_0 \mu_r$

$\mu = \mu_0 + K$

$\Rightarrow \mu_0 \mu_r = \mu_0 + K$

$\Rightarrow \mu_r = \dfrac{\mu_0 + K}{\mu_0}$

$\Rightarrow \mu_r = 1 + \dfrac{K}{\mu_0}$

Consider a solenoid having magnetic path of 'l' meter, area of cross section 'A' m^2 and a coil of 'N' turn carrying a current of 'I' Amp, then the magnetic field strength is

$$H = \frac{NI}{l} \quad N/wb$$

$$B = \mu H$$

$$\Rightarrow B = \frac{\mu NI}{l}$$

ϕ = flux

$$\Rightarrow \phi = BA$$

$$\Rightarrow \phi = \frac{\mu N I A}{l}$$

$$\Rightarrow \phi = \frac{\mu_0 \mu_r N I A}{l}$$

$$\Rightarrow \phi = \frac{NI}{\frac{l}{\mu_0 \mu_r A}} \quad wb.$$

'$\frac{l}{\mu_0 \mu_r A}$' is called reluctance (S).

'NI' is magneto motive force (mmf).

$$\phi = \frac{mmf}{Reluctance}$$

Magneto motive Force :-

→ It drives or tends to drive flux through a magnetic circuit like electromotive force (emf) in the electric circuit.

→ mmf is equal to the work done in Joules in carrying a unit magnetic pole once through the entire magnetic circuit.

→ It measured in Ampere-turn.

Ampere-turn (AT) :-

→ It is the unit of magnetomotive force (mmf) and is given by the product of number of turns in a magnetic ckt and the current in Ampere in those turns,

Reluctance :- (S)

It is the property of a material which opposes the creation of flux or it opposes the flow of magnetic flux through a material.

$$S = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}$$

$$\text{unit} = \frac{1}{\text{Henery}}$$

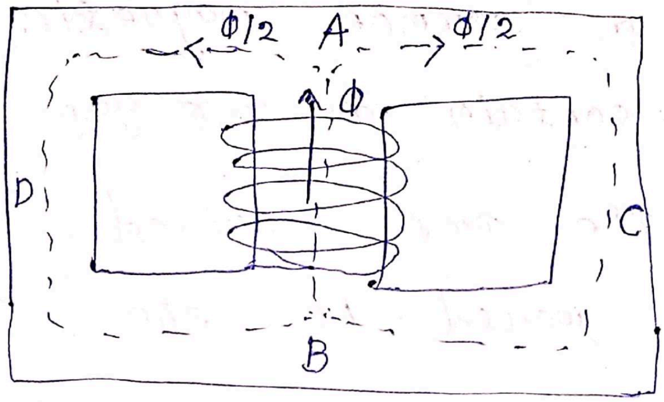
Permeance :-

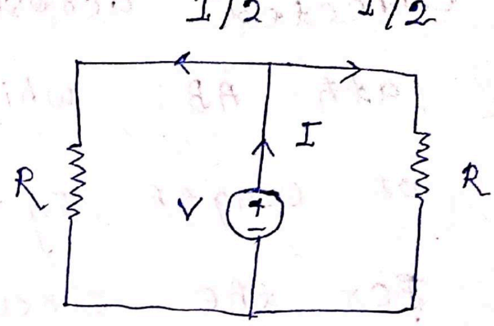
It is the reciprocal of reluctance or readiness for flow of magnetic flux.

Reluctivity :-

It is the specific Reluctance.

parallel magnetic circuit :-

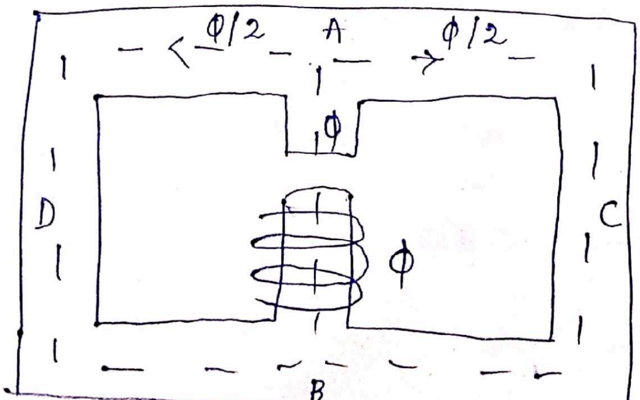


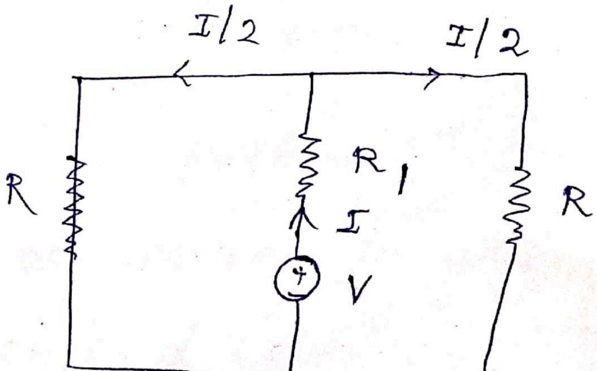


The above figure shows a parallel magnetic circuit consisting of two parallel magnetic path ACB and ADB given by same mmf.

The flux produced by the coil wound on a central core is divide equally at point 'A' between two outer parallel path.

Series parallel magnetic circuit :-

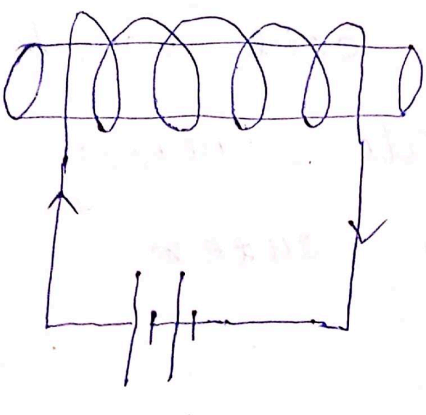




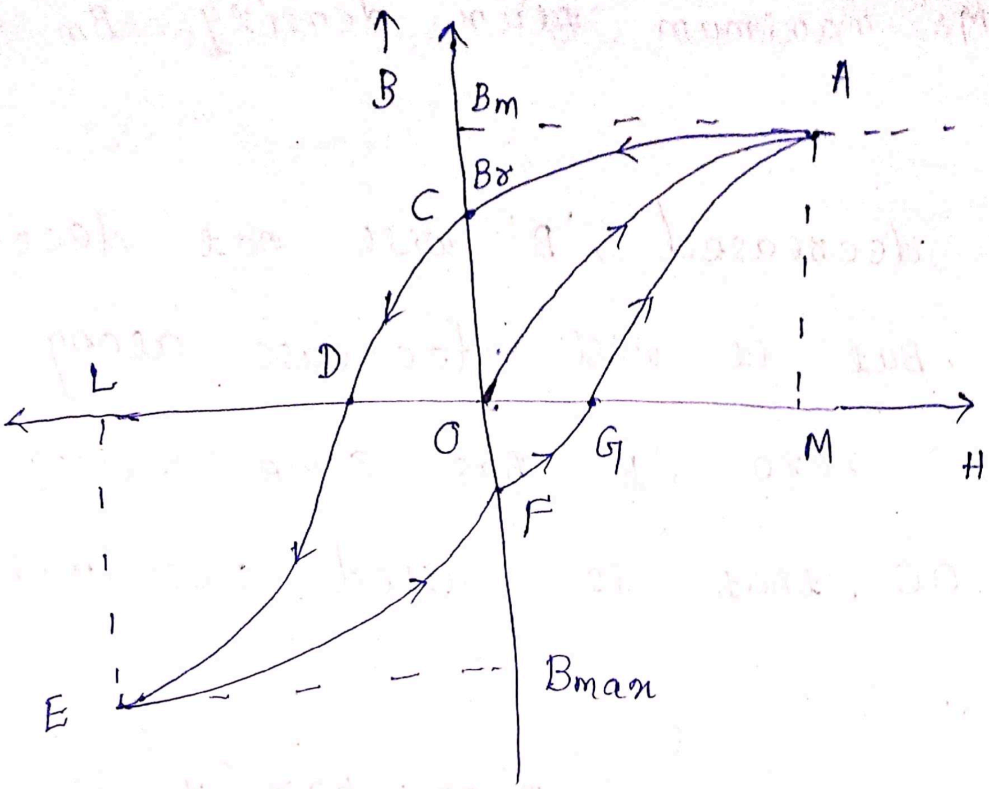
The above figure shows two parallel magnetic circuit ACB and ADB connected across a common magnetic path AB which contain an air gap of length I_g. The mmf required for the circuit would be the sum of,

(i) that required for the air gap.

(ii) that required for the parallel circuit.



B-H curve :-



Br = residual magnetism

→ It is defined as the lagging of magnetic flux density 'B' behind the magnetising force 'H'

Let us take an unmagnetised iron piece AB and magnetise it by placing within a field of solenoid. Then,

$$H = \frac{NI}{l}$$

Let 'H' is increase from 'O' to a maximum value then corresponding B value is noted.

If we plot the relation between H and B a curve like OA as shown in the above figure is obtained.

For $H = OM$ maximum flux density B_m is obtained.

→ If 'H' is decreased, 'B' will not decrease along AO. But it will decrease along AC.

→ When 'H' is zero, 'B' has some value i.e. $B_r = OC$, that is called residual magnetism.

→ To demagnetise the Iron bar 'H' is reversed. Then 'B' is reduced to zero at point 'D'. Where $H = OD$ and $B = 0$.

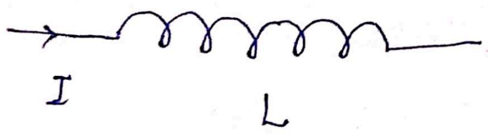
→ Again when 'H' value is further increased in -ve direction at point 'L', magnetic saturation is achieved.

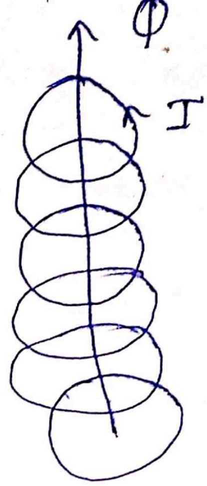
→ When 'H' is back from 'L' to 'M', a similar curve EFGA is obtained.

→ If we again start from 'G' the same curve GACDEFG is obtained once again.

→ The lagging of 'B' behind 'H' is known as Hysterisis.

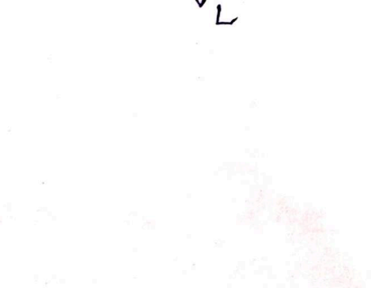
Coupled circuit





Self Inductance $\Rightarrow L = \dfrac{N\phi}{i}$

when a current changes in a circuit the magnetic flux linking the same circuit changes and an emf is induced in the circuit.



$$V_L = L\,\frac{di}{dt}$$

$$\Rightarrow V_L = L\,\frac{d}{dt}\left(\frac{N\phi}{L}\right)$$

$$\Rightarrow V_L = \cancel{L} \times \frac{N}{\cancel{L}} \times \frac{d\phi}{dt}$$

$$\Rightarrow V_L = N \frac{d\phi}{dt} = L \frac{di}{dt}$$

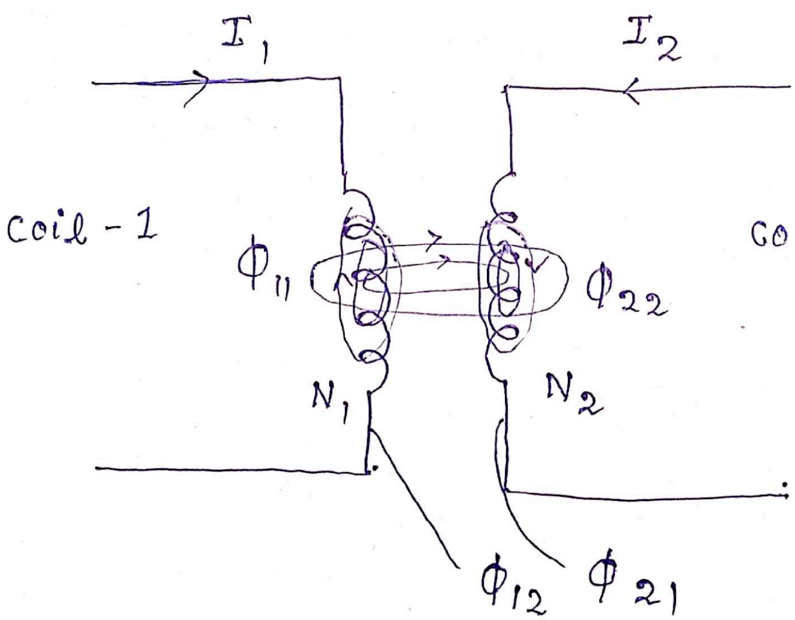
$$\Rightarrow N \frac{d\phi}{dt} = L \frac{di}{dt}$$

$$\Rightarrow N \, d\phi = L \, di$$

$$\Rightarrow \boxed{L = N \frac{d\phi}{dI}}$$

↳ self - Inductance.

Mutual Inductance :-



Let two coils carry a current of I_1 and I_2 (alternating current). Each coil have leakage flux of ϕ_{11} and ϕ_{22} and mutual flux of ϕ_{12} and

ϕ_{21} respectively.

$$V_{L_2} = \frac{N_2 \, d\phi_{12}}{dt}$$

$$\phi_{12} \propto I_1$$

$$\Rightarrow \phi_{12} = k I_1$$

$$= N_2 \frac{d \, k I_1}{dt}$$

$$V_{L_2} = N_2 k \frac{dI_1}{dt}$$

$$V_{L_2} = M \frac{dI_1}{dt}$$

$$M \frac{dI_1}{dt} = N_2 \frac{d\phi_{12}}{dt}$$

$$\Rightarrow M_2 = N_2 \frac{d\phi_{12}}{dI_1}$$

$$M_1 = N_1 \frac{d\phi_{21}}{dI_2}$$

$$\Rightarrow \boxed{\begin{aligned} M &= N_2 \frac{\phi_{12}}{I_1} \\ M &= N_1 \frac{\phi_{21}}{I_2} \end{aligned}}$$

$M \rightarrow$ Mutual - Inductance.

Co-efficient of coupling :-

It is defined as the fraction of total flux that links the coil.

k = co-efficient of coupling.

$$K = \frac{\phi_{12}}{\phi_1}$$

$$K = \frac{\phi_{21}}{\phi_2}$$

$$\Rightarrow \phi_{21} = K\phi_2$$

$$\Rightarrow \phi_{12} = K\phi_1$$

$$M = \frac{N_2 \phi_{12}}{I_1} \qquad M = \frac{N_1 \phi_{21}}{I_2}$$

$$M^2 = N_2 \frac{\phi_{12}}{I_1} \cdot N_1 \frac{\phi_{21}}{I_2}$$

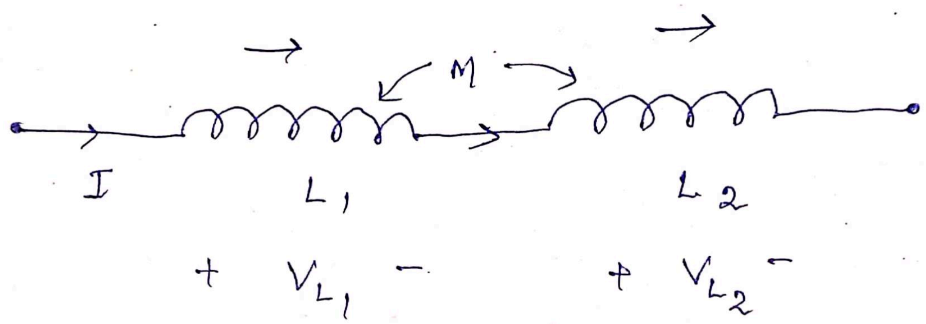
$$\Rightarrow M^2 = N_2 \frac{K\phi_1}{I_1} \cdot N_1 \frac{K\phi_2}{I_2}$$

$$\Rightarrow M^2 = K^2 \frac{N_1 \phi_1}{I_1} \cdot \frac{N_2 \phi_2}{I_2}$$

$\Rightarrow M^2 = K^2 L_1 L_2$

$\Rightarrow \boxed{M = K\sqrt{L_1 L_2}}$

series connection of coupled circuit :-



$$V_{L_1} = L_1 \frac{dI}{dt} + M\frac{dI}{dt}$$

$$= (L_1 + M)\frac{dI}{dt}$$

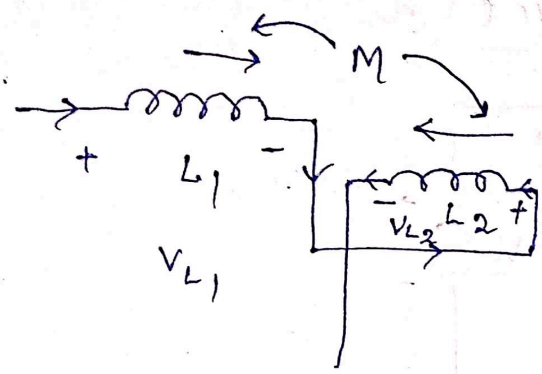
$$V_{L_2} = L_2 \frac{dI}{dt} + M\frac{dI}{dt}$$

$$= (L_2 + M)\frac{dI}{dt}$$

$$V_{L_1} + V_{L_2} = (L_1 + M)\frac{dI}{dt} + (L_2 + M)\frac{dI}{dt}$$

$$\Rightarrow \boxed{V_L = (L_1 + L_2 + 2M)\frac{dI}{dt}}$$

$$\boxed{L = L_1 + L_2 + 2M}$$



$$V_{L_1} = L_1 \frac{dI}{dt} - M \frac{dI}{dt}$$

$$= (L_1 - M) \frac{dI}{dt}$$

$$V_{L_2} = L_2 \frac{dI}{dt} - M \frac{dI}{dt}$$

$$= (L_2 - M) \frac{dI}{dt}$$

$$V_{L_1} + V_{L_2} = (L_1 - M) \frac{dI}{dt} + (L_2 - M) \frac{dI}{dt}$$

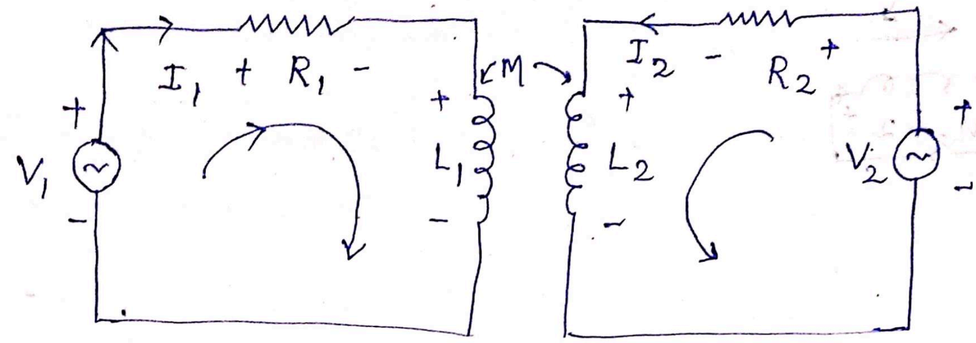
$$\Rightarrow V_L = (L_1 - M + L_2 - M) \frac{dI}{dt}$$

$$\Rightarrow \boxed{V_L = (L_1 + L_2 - 2M) \frac{dI}{dt}}$$

$$\Rightarrow \boxed{L = L_1 + L_2 - 2M}$$

Coupled circuit :-

Modeling of Coupled circuit :-



$$V_{L_1} = L_1 \frac{di}{dt}$$

$$L_{L_1} = M \frac{di_2}{dt}$$

$$V_1 - I_1 R_1 - L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} = 0$$

$$\Rightarrow V_1 = I_1 R_1 + L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt}$$

$$V_2 = I_2 R_2 + L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt}$$

$$V_1 = I_1 R_1 + j\omega L_1 I_1 - j\omega M I_2$$

$$V_2 = I_2 R_2 + j\omega L_2 I_2 - j\omega M I_1$$

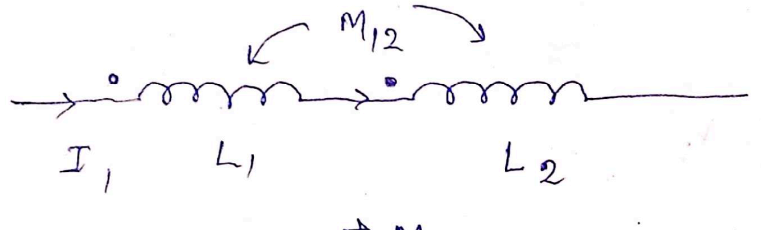
$$V_1 = (R_1 + j\omega L_1) I_1 - j\omega M I_2$$

$$V_2 = -j\omega M I_1 + (R_2 + j\omega L_2) I_2$$

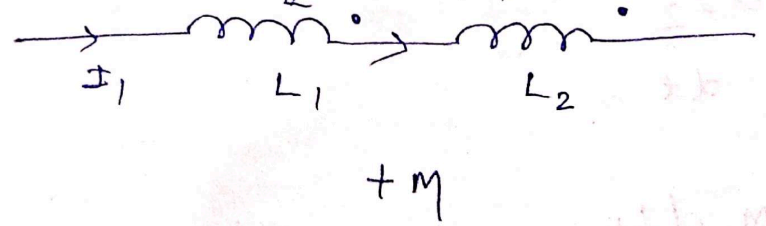
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} R_1 + j\omega L_1 & -j\omega M \\ -(j\omega M) & R_2 + j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Dot convention :-

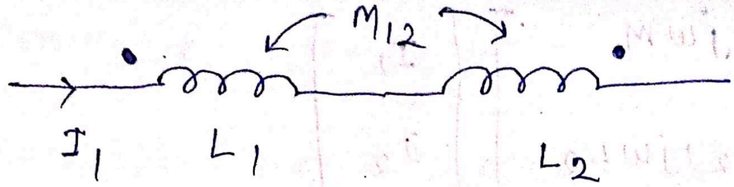
To determine the relative polarity of the induced voltage in the coupled coil, the coils are marked with dot. When current through each of the mutually coupled coils are going away from the dots or towards the dot, the mutual inductance is positive but when the current through the coils is leaving the dot for one coil and entering the other, the mutual inductance is negative.



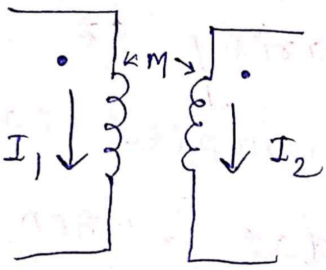
$+M$

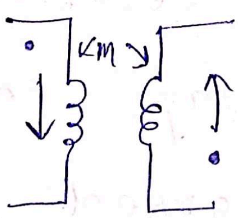


$+M$

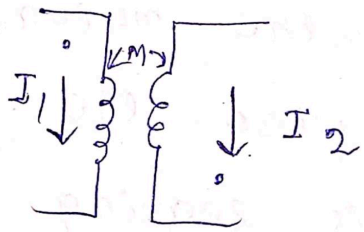


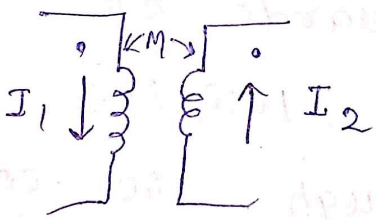
$-M$



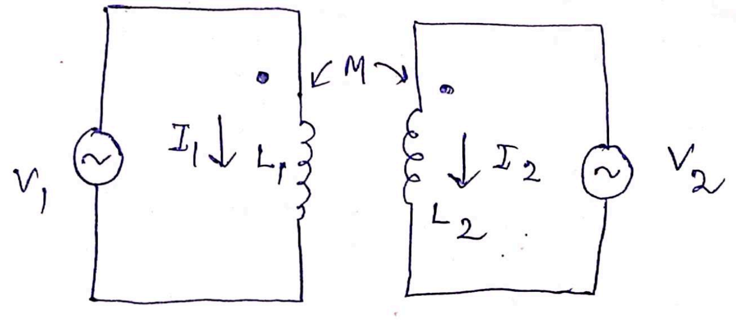


$(+M)$ $(+M)$



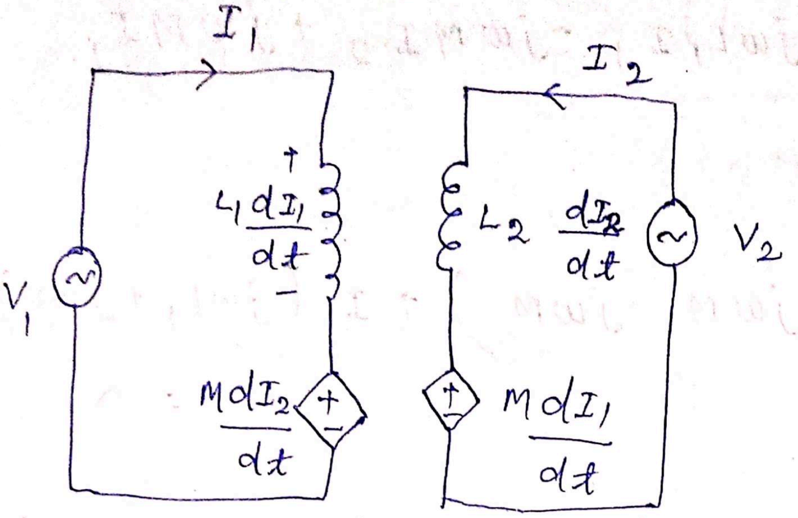


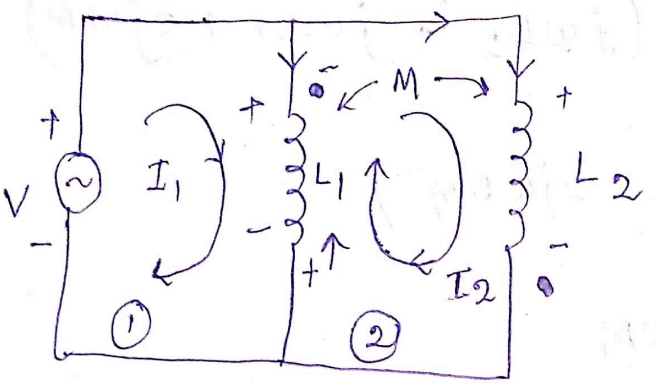
$(-M)$ $(-M)$



$$V_1 - L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt} = 0$$

$$V_2 - L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt} = 0$$





$$V_1 - L_1 \frac{d(I_1 - I_2)}{dt} + M \frac{dI_2}{dt} = 0$$

$$V_1 - j\omega L_1 (I_1 - I_2) + j\omega M I_2 = 0$$

$$V_1 = j\omega L_1 (I_1 - I_2) - j\omega M I_2$$

$$= j\omega L_1 I_1 - j\omega L_1 I_2 - j\omega M I_2$$

$$V_1 = j\omega L_1 I_1 - I_2 (j\omega L_1 + j\omega M) \quad \text{—— (1)}$$

$$-j\omega L_2 I_2 - j\omega L_1 (I_2 - I_1) - j\omega M (I_2 - I_1)$$
$$- j\omega M I_2 = 0$$

$$\Rightarrow -j\omega L_2 I_2 - j\omega L_1 I_2 + j\omega L_1 I_1 - j\omega M I_2 + j\omega M I_1$$

$$- j\omega M I_2 = 0$$

$$\Rightarrow I_2 (-j\omega L_2 - j\omega L_1 - j\omega M - j\omega M) + I_1 (j\omega L_1 + j\omega M) = 0$$

$$\Rightarrow I_1 (j\omega L_1 + j\omega M) = -I_2 (-j\omega L_2 - j\omega L_1 - 2j\omega M)$$

$$\Rightarrow I_1 (j\omega L_1 + j\omega M) = I_2 (j\omega L_2 + j\omega L_1 + 2j\omega M)$$

$$\Rightarrow I_1 = I_2 \frac{(j\omega L_2 + j\omega L_1 + 2j\omega M)}{j\omega L_1 + j\omega M}$$

$$\Rightarrow I_2 = \left(\frac{j\omega L_1 + j\omega M}{j\omega L_2 + j\omega L_1 + 2j\omega M} \right) I_1$$

$$\Rightarrow I_2 = \left(\frac{j\omega (L_1 + M)}{j\omega (L_2 + L_1 + 2M)} \right) I_1$$

putting value of I_2 in eq^n (1),

$$V_1 = j\omega L_1 I_1 - \frac{(L_1 + M)(j\omega L_1 + j\omega M)}{(L_2 + L_1 + 2M)} \times I_1$$

$$\Rightarrow V_1 = I_1 \left(j\omega L_1 - \frac{(L_1+M)(j\omega L_1 + j\omega M)}{L_2+L_1+2M} \right)$$

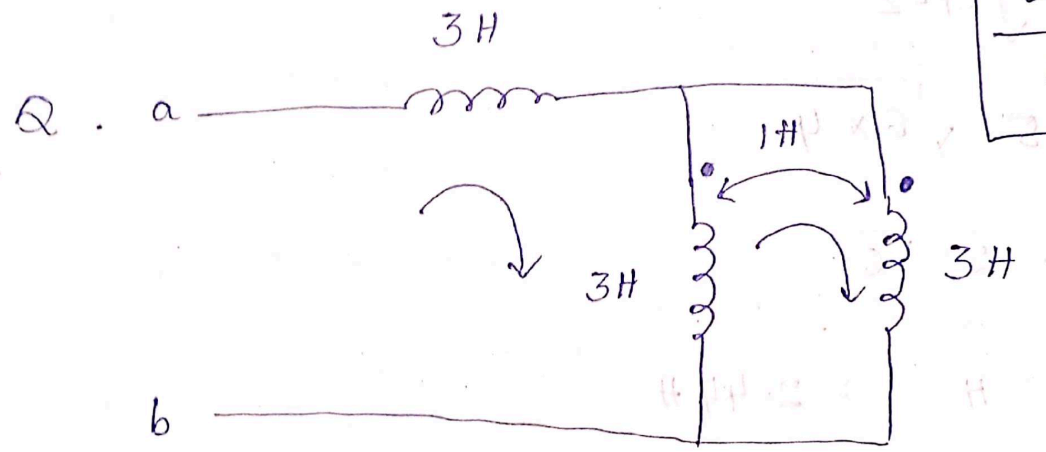
$$\Rightarrow \frac{V_1}{I_1} = j\omega \left(L_1 - \frac{(L_1^2 + L_1 M + M L_1 + M^2)}{L_2+L_1+2M} \right)$$

$$\Rightarrow \frac{V_1}{j\omega I_1} = \frac{L_1L_2 + \cancel{L_1^2} + \cancel{2ML_1} - \cancel{L_1^2} - \cancel{2ML_1} - M^2}{L_1+L_2+2M}$$

$$\Rightarrow L_{eq} = Z = \frac{V_1}{j\omega L_1} = \frac{L_1L_2 - M^2}{L_1+L_2+2M} \text{ (opposing)}$$

$$\boxed{\frac{L_1L_2-M^2}{L_1+L_2-2M} \text{ (adding)}}$$

Q.



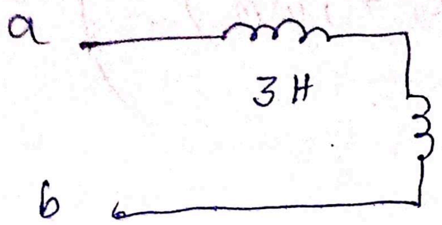
$L_1 = 3H$, $L_2 = 3H$, $M = 1H$

$$L = \frac{3\times 3 - 1^2}{3+3-2\times 1}$$

$$= \frac{9-1}{6-2}$$

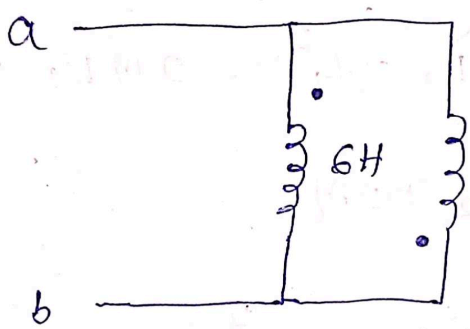
$$= \frac{8}{4}$$

$$= 2H$$



$L_{eq} = 3 + 2 = 5\ H$ (Ans)

Q.



$K = 0.5$

$L_{eq} =$

$M = K\sqrt{L_1 L_2}$

$= 0.5\sqrt{6 \times 4}$

$= \frac{1}{2} \cdot 2\sqrt{6}$

$= \sqrt{6}\ H = 2.44\ H$

$$L_{eq} = \frac{6 \times 4 - 6}{6 + 4 + 2\sqrt{6}} = \frac{24 - 6}{10 + 4.88}$$

$$= \frac{18}{10 + 2\sqrt{6}} = \frac{18}{14.88}$$

$$= \frac{9}{5 + \sqrt{6}}\ H = 1.2\ H \quad \text{(Ans)}$$

Q. Two coupled coil have self inductances $L_1 = 10 \times 10^{-3}$ H and $L_2 = 20 \times 10^{-3}$ H. The co-efficient of coupling $k = 0.75$. Find the voltage in the second coil and flux of the 1st coil provided the second coil has 500 turns and the circuit current is given by $I_1 = 2 \sin 314 t$ A.

Sol^n Given data,

$L_1 = 10 \times 10^{-3}$ H

$L_2 = 20 \times 10^{-3}$ H

$k = 0.75$

$V_2 = ?$

$I_1 = 2 \sin 314 t$

$$M = k\sqrt{L_1 L_2}$$

$$= 0.75\sqrt{10 \times 10^{-3} \times 20 \times 10^{-3}}$$

$$= 0.75\sqrt{200 \times 10^{-6}}$$

$$= 0.75\sqrt{2 \times 10^{-4}}$$

$$= 0.75 \sqrt{2} \times 10^{-2}$$

$$= 0.01 \text{ H}$$

$$V_2 = M \frac{dI_1}{dt}$$

$$= 0.01 \times \frac{d}{dt}\left(2\sin 314t\right)$$

$$= 0.01 \times 2 \frac{d}{dt} \sin(314t)$$

$$= 0.02 \times \cos 314t \times 314$$

$$= 6.28 \cos 314t$$

$$L_1 = \frac{N_1 \phi_1}{I_1}$$

$$M_2 = \frac{N_2 \phi_{12}}{I_1}$$

$$M_2 = \frac{N_2 (K\phi_1)}{I_1}$$

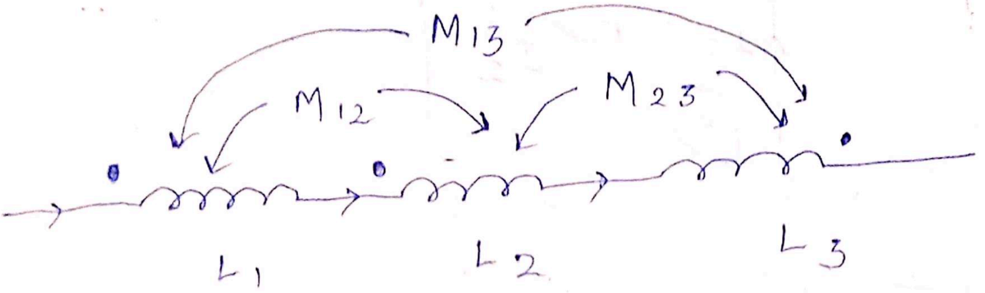
$$M_2 = M_1 = M$$

$$\Rightarrow 0.01 = \frac{500 \times (0.75) \times \phi_1}{2\sin 314t}$$

$$\Rightarrow \phi_1 = \frac{0.01 \times 2\sin 314t}{500 \times 0.75}$$

$\Rightarrow \phi_1 = 5.3 \times 10^{-5} \sin 314t$

Q. Find the total inductance of the three series connected coupled coil as shown in the figure.



$L_1 = 1H$

$L_2 = 6H$

$L_3 = 3H$

$M_{12} = 2H$

$M_{23} = 4H$

$M_{13} = 5H$

Sol^n

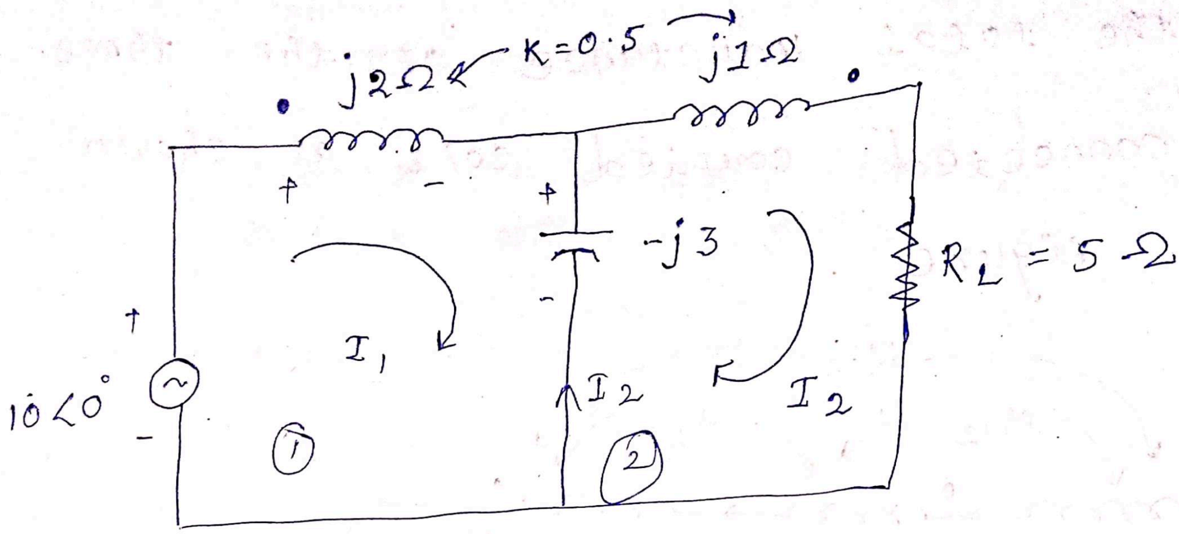
$L_1 + M_{12} - M_{13} + L_2 - M_{23} + M_{12} + L_3 - M_{23} - M_{13}$

$= 1 + 2 - 5 + 6 - 4 + 2 + 3 - 4 - 5$

$= 14 - 18$

$= -4\ H$

Q. Find the drop across R_L.



$M = K\sqrt{L_1 L_2}$

$= 0.5\sqrt{2 \times 1}$

$= 0.7\ H$

for loop (1),

$10 \angle 0 - I_1(j2) - (I_1 - I_2)(-j3)$

$10 \angle 0^\circ = I_1(j2) + (I_1 - I_2)(-j3) - I_2(0.707j)$ ——— (1)

for loop (2),

$0 - I_2(j1) - I_2 \times 5 - (I_2 - I_1)(-j3) = 0$

$\Rightarrow 0 = I_2(jI) + I_2 \times 5 + (I_2 - I_1)(-j3)$

$- I_1(0.707j)$

——— (2)

solving eq^n 1,

$$10 \angle 0^\circ = j2 I_1 - j3 I_1 + j3 I_2 - j\,0.707\, I_2$$

$$10 \angle 0^\circ = -jI_1 + 2.293 j I_2$$

solving eq^n 2,

$$0 = j1 I_2 + 5I_2 - j3 I_2 + j3 I_1 - j\,0.707\, I_1$$

$$0 = -j2I_2 + 5I_2 + j\,2.293\, I_1$$

$$0 = I_2(5 - j2) + j\,2.293\, I_1$$

$$10 \angle 0^\circ = -jI_1 + j\,2.293 I_2 \quad \text{--- (3)}$$

$$0 = j\,2.293\, I_1 + (5 - j2)\, I_2 \quad \text{--- (4)}$$

$$\begin{bmatrix} 10 \angle 0^\circ \\ 0 \end{bmatrix} = \begin{bmatrix} -j1 & j2.293 \\ j2.293 & 5-j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$I_2 = \frac{\begin{bmatrix} -j1 & 10 \angle 0^\circ \\ j2.293 & 0 \end{bmatrix}}{\begin{bmatrix} -j1 & j2.293 \\ j2.293 & 5-j2 \end{bmatrix}}$$

$$I_2 = \frac{\left(0 - 10\angle 0^\circ \left(+j2.293\right)\right)}{-j1(5-j2) - (j2.293)(j2.293)}$$

$$= \frac{\left(0 - 10\angle 0 \times \left(2.293\angle 90^\circ\right)\right)}{\left(1\angle -90^\circ\right)\left(5.385\angle -21.801^\circ\right) - \left(2.293\angle 90^\circ\right)\left(2.293\angle 90^\circ\right)}$$

$$= \frac{\left(0 - 10\angle 0^\circ \left(0 + j\,2.293\right)\right)}{(0-j1)(5-j2) - (0+j\,2.293)(0+j\,2.293)}$$

$$\begin{bmatrix} 10\angle 0^\circ \\ 0 \end{bmatrix} = \begin{bmatrix} -j1 & j2.293 \\ j2.293 & 5-j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10+j0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j1 & j2.293 \\ j2.293 & 5-j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{bmatrix} 10+j0 & j2.293 \\ j2.293 & 5-j2 \end{bmatrix}}{\begin{bmatrix} -j1 & j2.293 \\ j2.293 & 5-j2 \end{bmatrix}}$$

$$= \frac{35.257849 - j20}{3.257849 - j5}$$

$$= \frac{58.765 \angle -19.897}{5.967 \angle -56.91}$$

$$= 9.8483 \angle 37.01$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{bmatrix} -j1 & 10+j0 \\ j2.293 & 0 \end{bmatrix}}{\begin{bmatrix} -j1 & j2.293 \\ j2.293 & 5-j2 \end{bmatrix}}$$

$$= \frac{[AB - CD]}{[AB - CD]}$$

$$= \frac{0 - 22.93j}{3.257849 - 5i}$$

$$= \frac{22.93 \angle -90}{5.9677711463 \angle -56.912903}$$

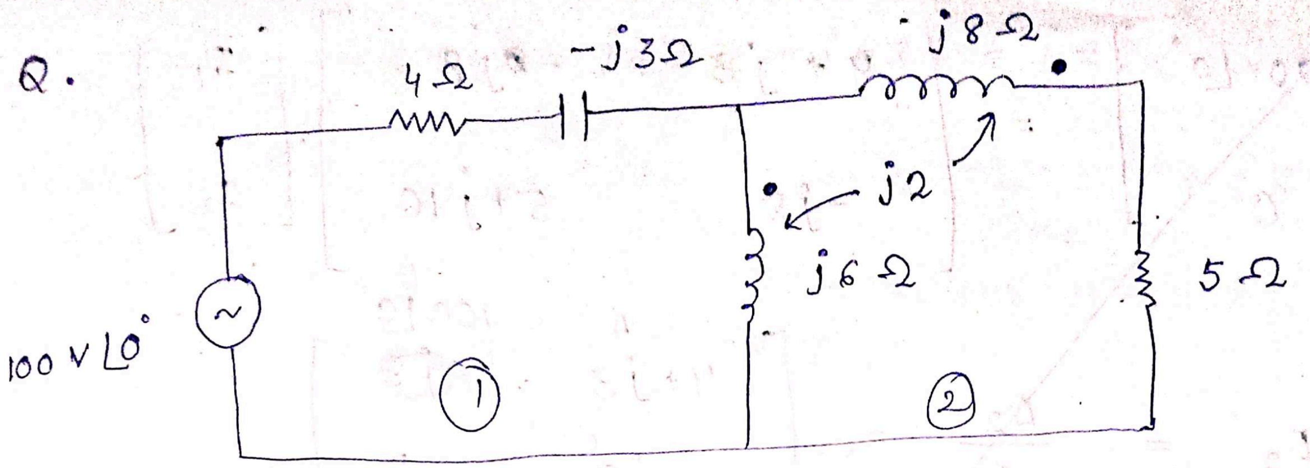
$$= 3.842344 \angle -33.0871^\circ$$

$$V = I_2 R_L$$

$$= (3.842 \angle -33.087)(5 \angle 0)$$

$$= 19.21 \angle -33.087 \text{ V}$$

Q.



Determine the voltage drop across the 5 Ω resistor.

Sol

$$100\angle 0^\circ = 4I_1 + (-j3)I_1 + j6(I_1 - I_2) - j2I_2$$

$$\Rightarrow 100\angle 0^\circ = 4I_1 - j3I_1 + j6I_1 - j6I_2 - j2I_2$$

$$\Rightarrow 100\angle 0^\circ = I_1(4+j3) - 8jI_2$$

$$\Rightarrow 100\angle 0^\circ = (4+j3)I_1 - 8jI_2 \quad \text{—(1)}$$

for loop (2),

$$0 = j8I_2 + 5I_2 + j6(I_2 - I_1) + j2(I_2 - I_1) + j2I_2$$

$$\cancel{\Rightarrow 0 = j8I_2 + 5I_2 + j6I_2 - j6I_1 + j2I_2 - j2I_1}$$

$$\cancel{\Rightarrow 0 = -j8I_1 + (5+16j)I_2} \quad \text{—(2)}$$

$$0 = j8 I_2 + 5 I_2 + j6 I_2 - j6 I_1 + j2 I_2 - j2 I_1 + j2 I_2$$

$$\Rightarrow 0 = j18 I_2 + 5 I_2 - j8 I_1$$

$$\Rightarrow 0 = -j8 I_1 + (5 + j18) I_2 \quad \text{—— (2)}$$

$$\begin{bmatrix} 100\angle 0^\circ \\ 0 \end{bmatrix} = \begin{bmatrix} 4+j3 & -j8 \\ -j8 & 5+j18 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow I_2 = \frac{\Delta_2}{\Delta_0} = \frac{\begin{bmatrix} \overset{A}{4+j3} & \overset{D}{100\angle 0^\circ} \\ \overset{C}{-j8} & \overset{B}{0} \end{bmatrix}}{\begin{bmatrix} \overset{E}{4+j3} & \overset{Y}{-j8} \\ \overset{X}{-j8} & \overset{F}{5+j18} \end{bmatrix}}$$

$$= \frac{AB - CD}{EF - XY}$$

$$= \frac{j800}{30 + j87}$$

$$= 8.2182 + j2.8338$$

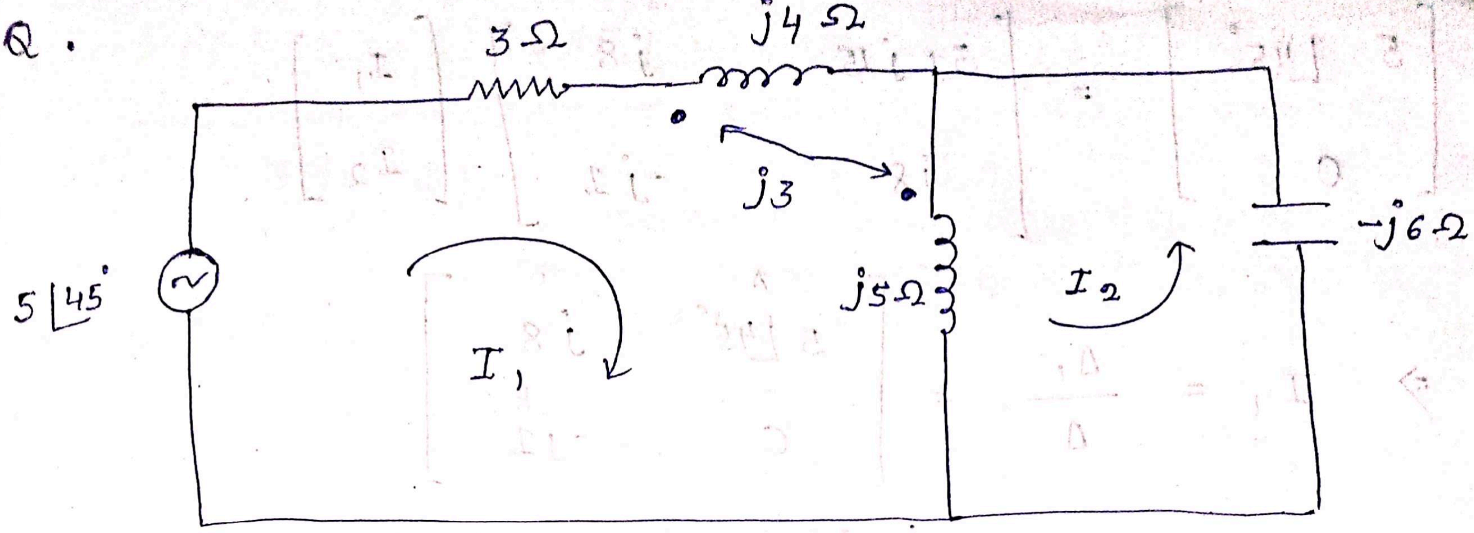
$$= 8.69305 \angle 19.025^\circ$$

$$V = I_2 \times 5$$

$$= (8.69305 \angle 19.025^\circ)(5 \angle 0^\circ)$$

$$= 43.465 \angle 19.025^\circ$$

Q.



for loop ①,

$$5\angle 45^\circ = 3I_1 + j4I_1 + j5(I_1 + I_2) + j3(I_1 + I_2) + j3I_1$$

$$\Rightarrow 5\angle 45^\circ = 3I_1 + j4I_1 + j5I_1 + j5I_2 + j3I_1 + j3I_2 + j3I_1$$

$$\Rightarrow 5\angle 45^\circ = 3I_1 + 15jI_1 + 8jI_2$$

$$\Rightarrow 5\angle 45^\circ = (3 + 15j)I_1 + 8jI_2 \quad \text{—— } (1)$$

$$\Rightarrow 5\angle 45^\circ = (3 + j15)I_1 + j8I_2 \quad \text{—— } (1)$$

for loop ②,

$$0 = -j6I_2 + j5(I_2 + I_1) + j3I_1$$

$$\Rightarrow 0 = -j6I_2 + j5I_2 + j5I_1 + j3I_1$$

$$\Rightarrow 0 = j8I_1 - j1I_2 \quad \text{—— } (2)$$

$$\begin{bmatrix} 5\angle 45^\circ \\ 0 \end{bmatrix} = \begin{bmatrix} 3+j15 & j8 \\ j8 & -j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow \quad I_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{bmatrix} \overset{A}{5\angle 45^\circ} & \overset{D}{j8} \\ \overset{C}{0} & \overset{B}{-j1} \end{bmatrix}}{\begin{bmatrix} \overset{E}{3+j15} & \overset{Y}{j8} \\ \overset{X}{j8} & \overset{F}{-j1} \end{bmatrix}}$$

$$= \frac{AB - CD}{EF - XY}$$

$$= \frac{3.5355 - 3.5355j}{79 - 3j}$$

$$= 0.04638 - 0.04299j$$

$$= 0.0632 \angle -42.827^\circ$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{bmatrix} \overset{A}{3 + j15} & \overset{D}{5\angle 45^\circ} \\ \overset{C}{j8} & \overset{B}{0} \end{bmatrix}}{\begin{bmatrix} \overset{E}{3 + j15} & \overset{Y}{j8} \\ \overset{X}{j8} & \overset{F}{-j1} \end{bmatrix}}$$

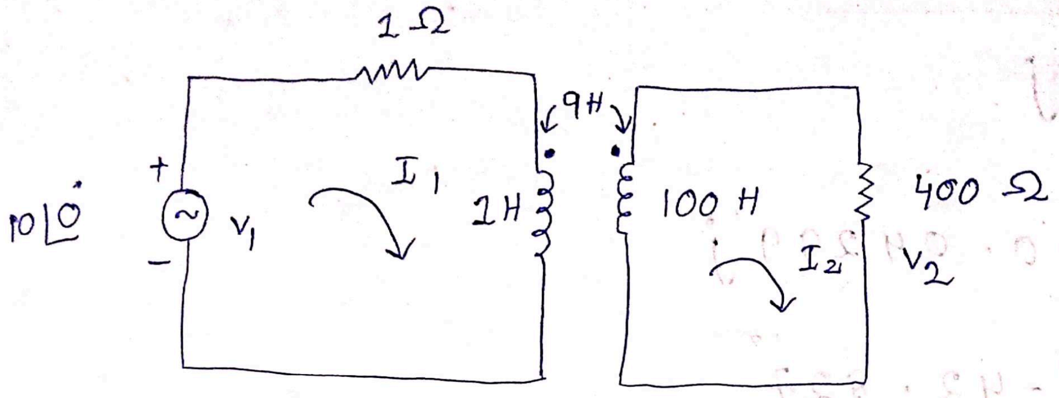
$$= \frac{AB - CD}{EF - XY}$$

$$= \frac{28.2842 - 28.2842j}{79 - 3j}$$

$$= 0.37108 - 0.34393j$$

$$= 0.50595 \angle -42.8254^\circ$$

Q.



Calculate the ratio of the output voltage across 400 Ω resister to the source voltage for the circuit shown in the figure.

$\omega = 10$ rad/sec

Sol^n

$$\frac{V_2}{V_1} = \frac{I_2 \times 400}{V_1} = ?$$

$L_1 = 1 \text{ H}$

$X_{L_1} = \omega L_1$

$= 10 \times 1$

$= 10\ \Omega$

$L_2 = 100 \text{ H}$

$X_{L_2} = \omega L_2$

$= 1000\ \Omega$

$X_M = \omega M = 10 \times 9 = 90\ \Omega$

for loop ①,

$$10\angle 0^\circ = 1 \times I_1 + j10 I_1 - j90 I_2$$

$$\Rightarrow 10\angle 0^\circ = (1 + j10) I_1 - j90 I_2 \quad — ①$$

for loop ②,

$$0 = 400 I_2 + j1000 I_2 - j90 I_1$$

$$\Rightarrow 0 = -j90 I_1 + (400 + j1000) I_2 \quad — ②$$

$$\begin{bmatrix} 10\angle 0^\circ \\ 0 \end{bmatrix} = \begin{bmatrix} 1+j10 & -j90 \\ -j90 & 400+j1000 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow I_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{bmatrix} 1+j10 & 10\angle 0^\circ \\ -j90 & 0 \end{bmatrix}}{\begin{bmatrix} 1+j10 & -j90 \\ -j90 & 400+j1000 \end{bmatrix}}$$

$$= \frac{j900}{-1500 + j5000}$$

$$= 0.1651376 - j0.0495412$$

$$= 0.1724086 \angle -16.699218^\circ$$

$$\frac{V_2}{V_1} = \frac{I_2 \times 400}{V_1}$$

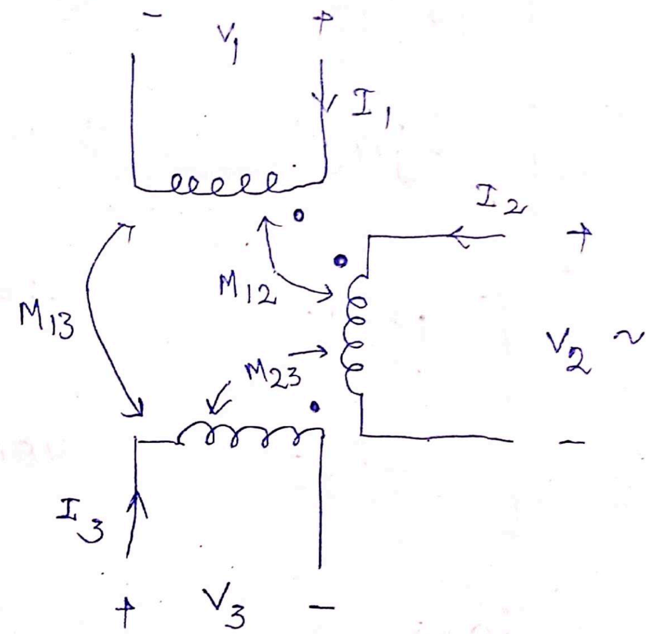
$$= \frac{(0.1724086 \angle -16.699218^\circ) \times 400}{10 \angle 0^\circ}$$

$$= \frac{68.96344 \angle -16.699218^\circ}{10 \angle 0^\circ}$$

$$= 6.6055004 6246 - j\, 1.9816468 4208$$

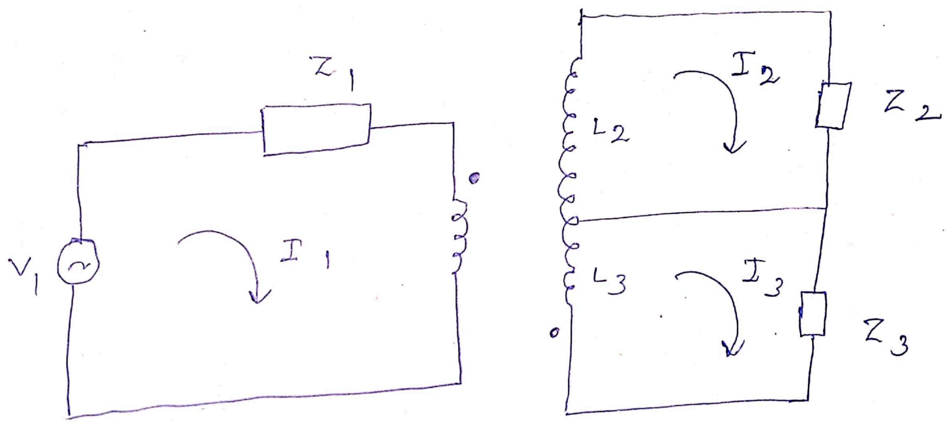
$$= 6.896344 \angle -16.699218^\circ \quad (\text{Ans})$$

Q.



find V_1, V_2 and V_3 ?

Q.

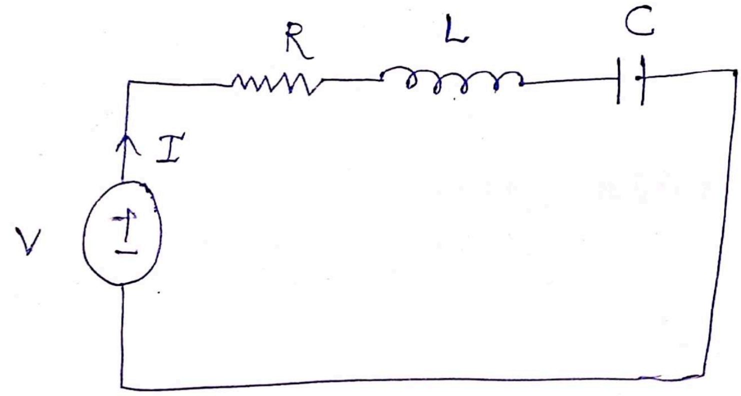


Find the mesh equation.

Circuit element and analysis :-

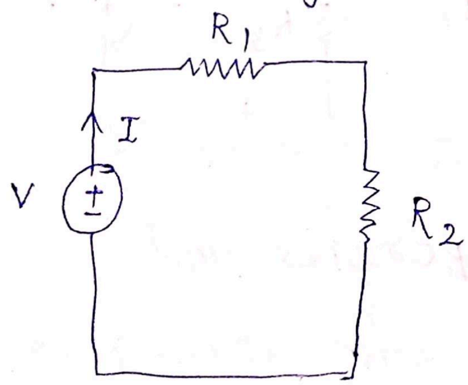
Circuit

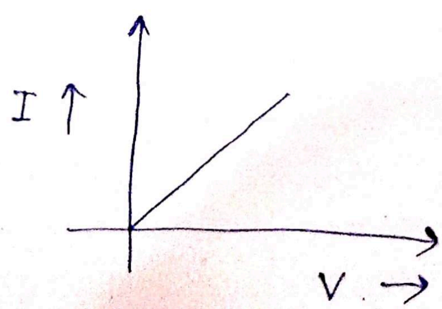
Circuit is a closed conducting path through which an electric current flows. It consists of active and passive element.



Linear circuit :-

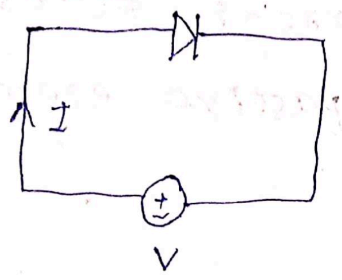
A linear circuit is one whose parameters are constant with time and they do not change with voltage or current and the circuit obeys ohm's law.

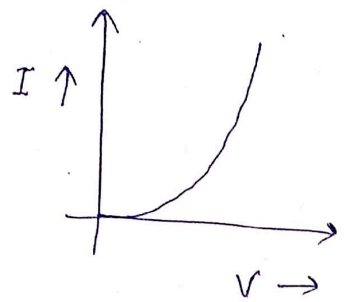




Non-linear circuit :-

It is that circuit whose parameters changes with voltage or current.

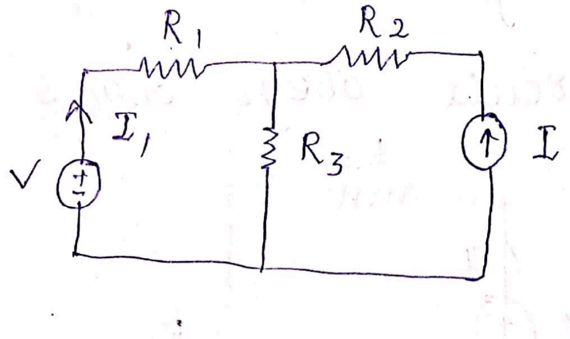




Bilateral Network :-

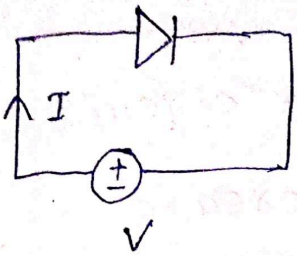
A bilateral circuit is one whose properties and characteristics are the same in either direction.

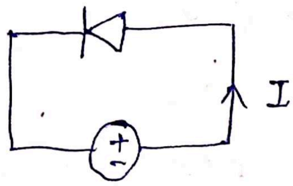
Ex :- Transmission line



Unilateral Network :-

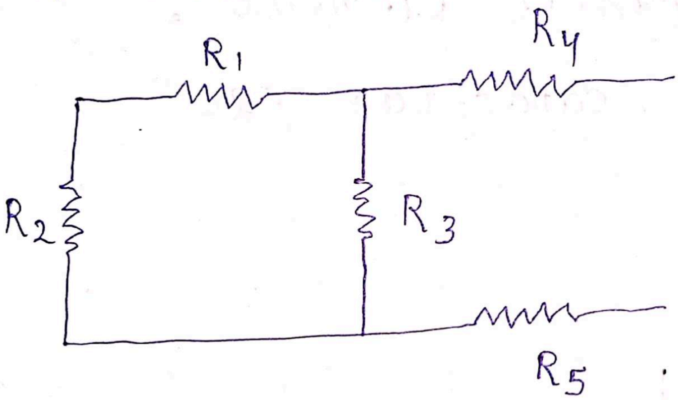
It is the circuit whose properties and characteristics changes with the direction of its operation.





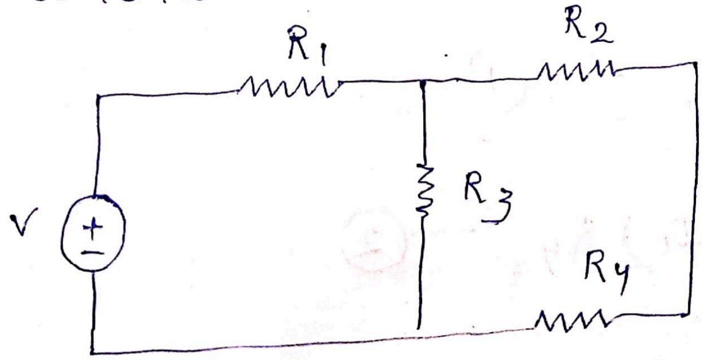
passive Network :-

It is the network which contain no source of emf .



Active Network :-

It is a network which contain one or more than source of emf with passive element .



Active element :-

An active element is defined as the circuit component which can increase or decrease the energy level of a signal passing through it.

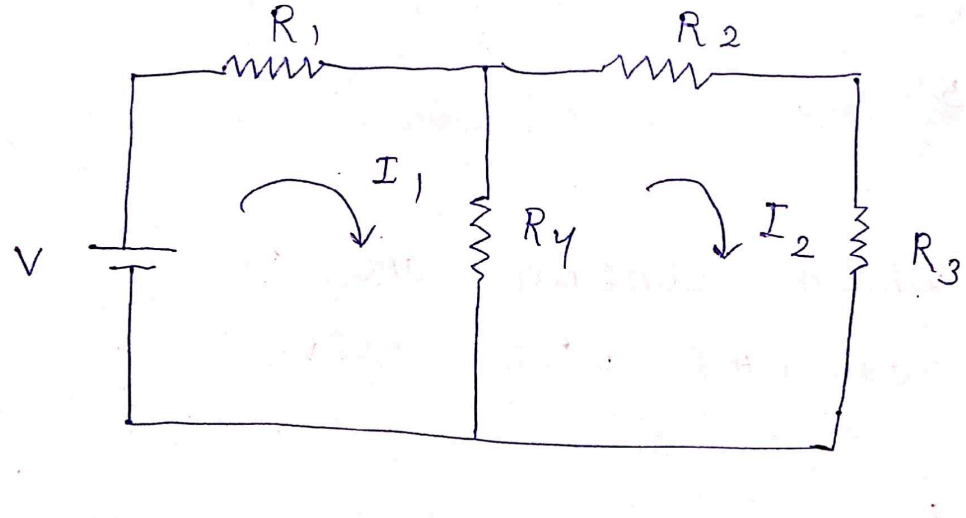
Ex :- Transistor, Op-Amp, Battery etc.

Passive element :-

Any circuit element which only consumes the power is called passive element.

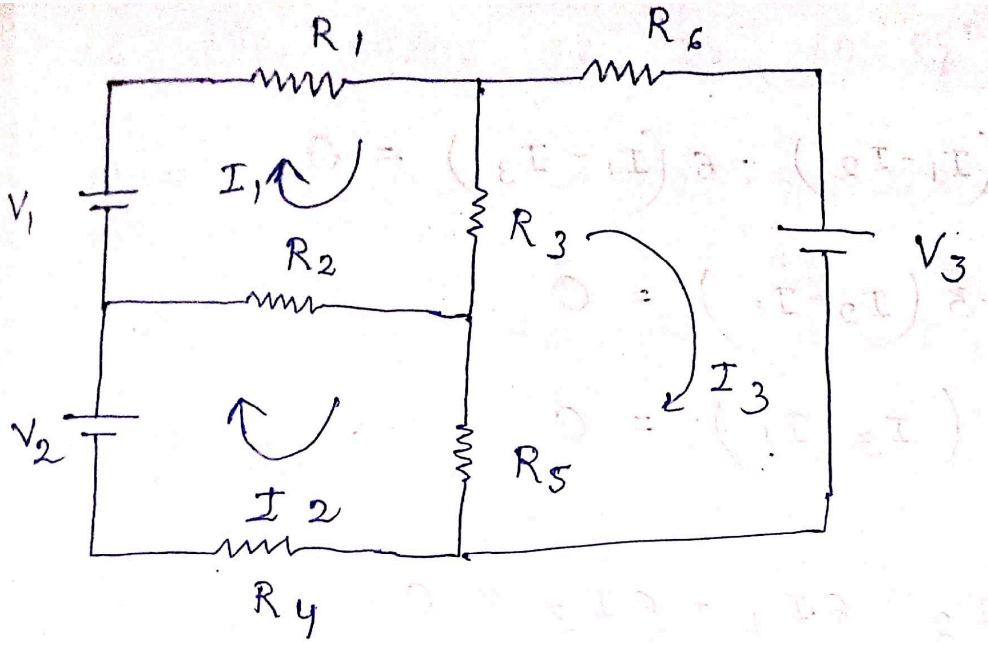
Ex :- Resistor, Inductor, capacitor etc.

Mesh analysis :-



$$V = I_1 R_1 + (I_1 - I_2) R_4 \quad \text{—— (1)}$$

$$0 = I_2 R_2 + I_2 R_3 + (I_2 - I_1) R_4 \quad \text{—— (2)}$$

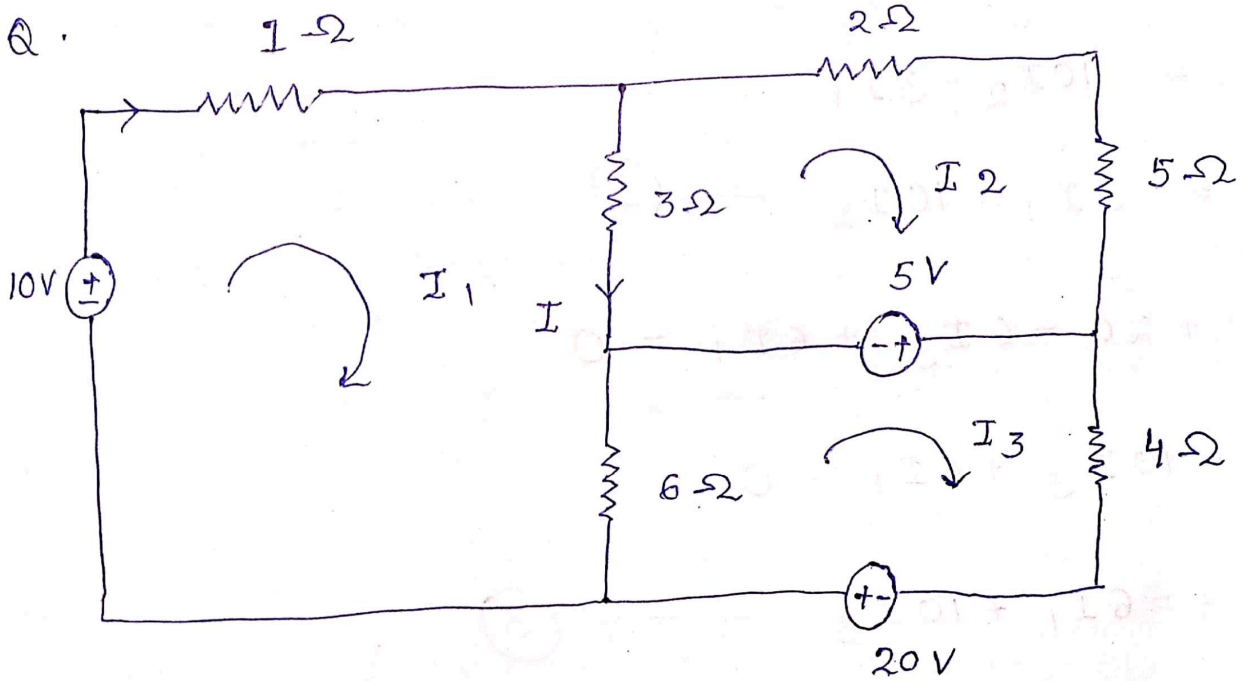


$$V_1 = I_1 R_1 + (I_1 - I_3) R_3 + (I_1 - I_2) R_2$$

$$V_2 = (I_2 - I_1) R_2 + (I_2 - I_3) R_5 + I_2 R_4$$

$$-V_3 = (I_3 - I_2) R_5 + (I_3 - I_1) R_3 + I_3 R_6$$

Q.



Determine current I using mesh analysis.

Sol^n

$$10 - I_1 \times 1 - 3(I_1 - I_2) - 6(I_1 - I_3) = 0$$

$$-2I_2 - 5I_2 - 5 - 3(I_2 - I_1) = 0$$

$$5 - 4I_3 + 20 - 6(I_3 - I_1) = 0$$

$$10 - I_1 - 3I_1 + 3I_2 - 6I_1 + 6I_3 = 0$$

$$10 - 10I_1 + 3I_2 + 6I_3 = 0$$

$$10 = 10I_1 - 3I_2 - 6I_3 \quad \text{—— (1)}$$

$$-2I_2 - 5I_2 - 5 - 3I_2 + 3I_1 = 0$$

$$\Rightarrow -10I_2 + 3I_1 - 5 = 0$$

$$\Rightarrow -5 = 10I_2 - 3I_1$$

$$\Rightarrow 5 = 3I_1 - 10I_2 \quad \text{—— (2)}$$

$$5 - 4I_3 + 20 - 6I_3 + 6I_1 = 0$$

$$\Rightarrow 25 - 10I_3 + 6I_1 = 0$$

$$\Rightarrow 25 = -6I_1 + 10I_3 \quad \text{—— (3)}$$

$$\Rightarrow 25 + 6I_1 = 10I_3$$

$$\Rightarrow I_3 = \frac{25 + 6I_1}{10}$$

$$\Rightarrow I_3 = 2.5 + 0.6I_1$$

put the value of I_3 in eq^n (1),

$$10 = 10I_1 - 3I_2 - 6(2.5 + 0.6I_1)$$

$$\Rightarrow 10 = 10I_1 - 3I_2 - 15 - 3.6I_1$$

$$\Rightarrow 10 + 15 = 6.4I_1 - 3I_2$$

$$\Rightarrow [25 = 6.4I_1 - 3I_2] \times 10 \quad \text{—— (4)}$$

$$[5 = 3I_1 - 10I_2] \times 3$$

$$\Rightarrow 250 = 64I_1 - 30I_2 \quad \text{—— (5)}$$

$$15 = 9I_1 - 30I_2 \quad \text{—— (6)}$$

substracting eq^n (6) from eq^n (5),

$$235 = 55I_1$$

$$\Rightarrow I_1 = 4.272$$

$$15 = 9(4.272) - 30I_2$$

$$\Rightarrow 30I_2 = 23.448$$

$$\Rightarrow I_2 = \frac{23.448}{30}$$

$$= 0.7816$$

$$I = I_1 - I_2$$

$$= 4.272 - 0.7816$$

$$= 3.4904 \text{ Amp}$$

$$\begin{bmatrix} 10 \\ 5 \\ 25 \end{bmatrix} = \begin{bmatrix} 10 & -3 & -6 \\ 3 & -10 & 0 \\ -6 & 0 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta}$$

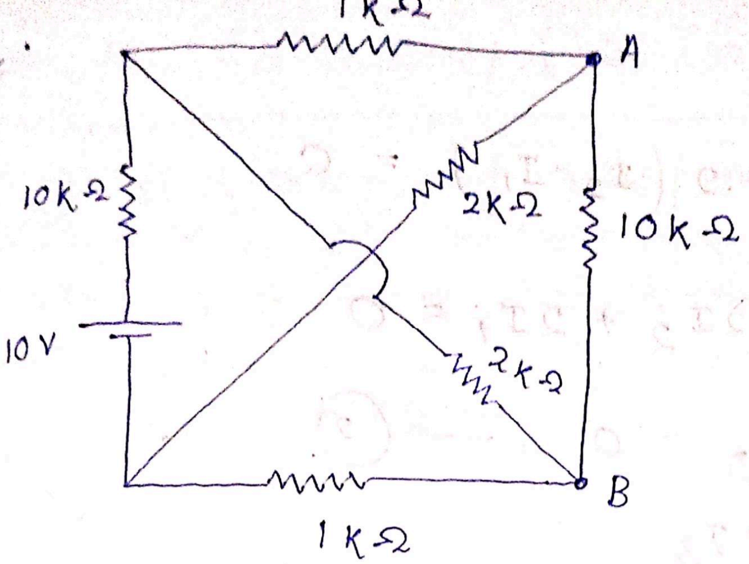
$$= \frac{\begin{bmatrix} 10 & -3 & -6 \\ 5 & -10 & 0 \\ 25 & 0 & 10 \end{bmatrix}}{\begin{bmatrix} 10 & -3 & -6 \\ 3 & -10 & 0 \\ -6 & 0 & 10 \end{bmatrix}}$$

$$= \underline{10\,(-100) + 3\,(50 - 0) - 6\,(0 + 250)}$$

$$= \underline{-1000 + 150 - 1500}$$

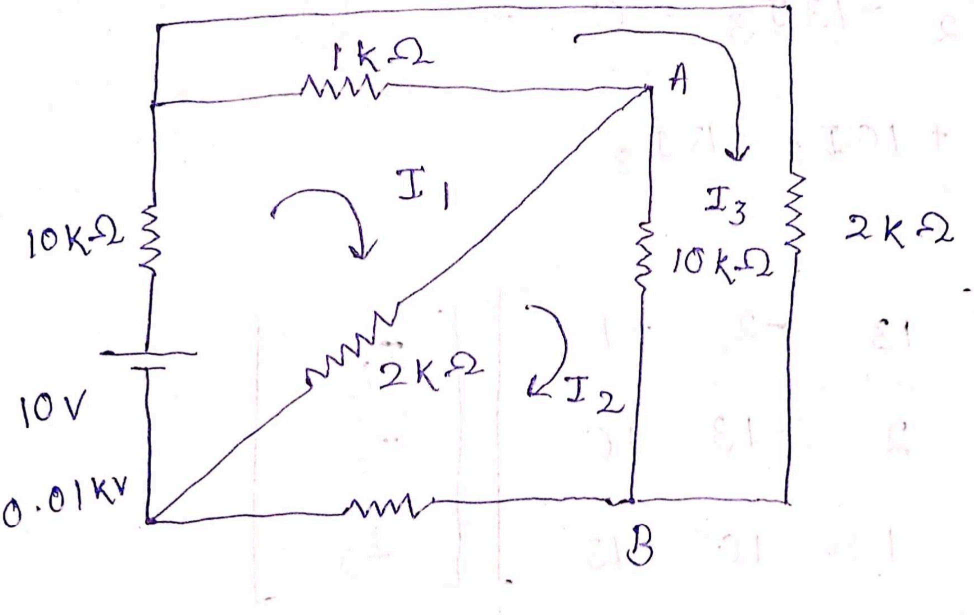
$$= \underline{-2350}$$

Q.



find the voltage across 10kΩ resistor at terminal AB as shown in the figure.

Sol^n



for loop (1),

$$0.01 - 10I_1 - 1(I_1 - I_3) - 2(I_1 - I_2) = 0$$

$$\Rightarrow 0.01 - 10I_1 - I_1 + I_3 - 2I_1 + 2I_2 = 0$$

$$\Rightarrow 0.01 - 13I_1 + 2I_2 + I_3 = 0$$

$$\Rightarrow 0.01 = 13I_1 - 2I_2 - I_3 \quad \text{—— (1)}$$

for loop (2),

$$-10(I_2 - I_3) - I_2 \cdot 1 - 2(I_2 - I_1) = 0$$

$$\Rightarrow -10I_2 + 10I_3 - I_2 - 2I_2 + 2I_1 = 0$$

$$\Rightarrow 2I_1 - 13I_2 + 10I_3 = 0 \quad \text{——— (2)}$$

$$\Rightarrow 0 = 2I_1 - 13I_2 + 10I_3$$

for loop (3)

$$-2I_3 - 10(I_3 - I_2) - 1(I_3 - I_1) = 0$$

$$\Rightarrow -2I_3 - 10I_3 + 10I_2 - I_3 + I_1 = 0$$

$$\Rightarrow I_1 + 10I_2 - 13I_3 = 0 \quad \text{——— (3)}$$

$$\Rightarrow 0 = I_1 + 10I_2 - 13I_3$$

$$\begin{bmatrix} 0.01 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 13 & -2 & -1 \\ 2 & -13 & 10 \\ 1 & 10 & -13 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{bmatrix} 13 & 0.01 & -1 \\ 2 & 0 & 10 \\ 1 & 0 & -13 \end{bmatrix}}{\begin{bmatrix} 13 & -2 & -1 \\ 2 & -13 & 10 \\ 1 & 10 & -13 \end{bmatrix}}$$

$$= \frac{13(0) - 0.01(-26-10) - 1(0)}{13(169-100) + 2(-26-10) - 1(20+13)}$$

$$= \frac{0 + 0.36 - 0}{897 - 72 - 33}$$

$$= \frac{0.36}{792}$$

$$= \frac{4.545}{10000}$$

$= 0.0004545$ KAmp

$= 0.4545$ Amp

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{\begin{bmatrix} 13 & -2 & 0.01 \\ 2 & -13 & 0 \\ 1 & 10 & 0 \end{bmatrix}}{\begin{bmatrix} 13 & -2 & -1 \\ 2 & -13 & 10 \\ 1 & 10 & -13 \end{bmatrix}}$$

$$= \frac{13(0) + 2(0) + 0.01(20+13)}{792}$$

$$= \frac{0.33}{792}$$

$= 0.00041$ KA

$= 0.41$ Amp

$V = IR$

$= (I_2 - I_3)\ 10000$

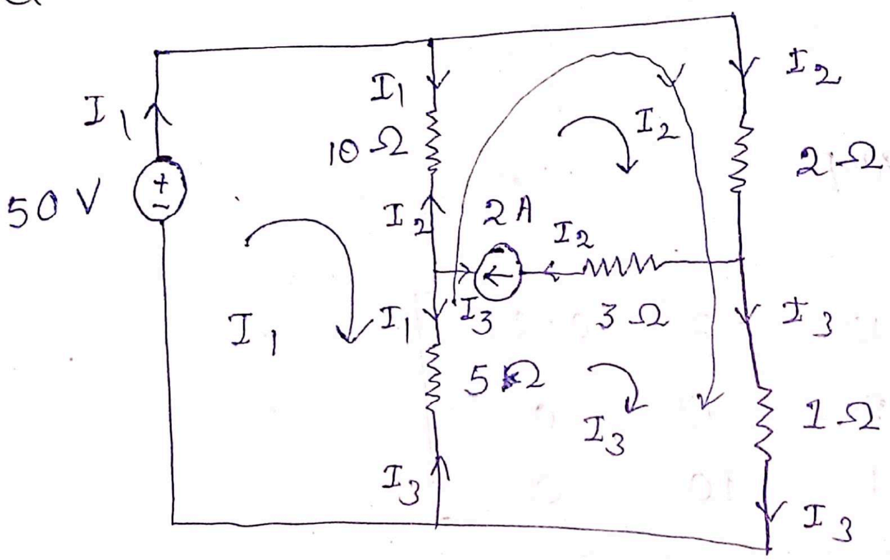
$= (0.045 - 0.41) \times 10000$

$= 0.04 \times 1000$

$= 400$ volt

Super Mesh Analysis :-

Q.



What is the voltage across 1 Ω resistor.

$$50 - 10(I_1 - I_2) - 5(I_1 - I_3) = 0$$

$$\Rightarrow 50 - 10I_1 + 10I_2 - 5I_1 + 5I_3 = 0$$

$$\Rightarrow 50 - 15I_1 + 10I_2 + 5I_3 = 0$$

$$\Rightarrow 50 = 15I_1 - 10I_2 - 5I_3 \quad \text{——— (1)}$$

$$-2I_2 - 1I_3 - 5(I_3 - I_1) - 10(I_2 - I_1) = 0$$

$$\Rightarrow -2I_2 - I_3 - 5I_3 + 5I_1 - 10I_2 + 10I_1 = 0$$

$$\Rightarrow 15I_1 - 12I_2 - 6I_3 = 0$$

$$\Rightarrow 0 = 15I_1 - 12I_2 - 6I_3 \quad \text{——— (2)}$$

$I_2 + 2 - I_3 = 0$

$\Rightarrow 2 = I_3 - I_2$ —— (3)

$\Rightarrow 2 = -I_2 + I_3$ —— (3)

$\Rightarrow I_3 = 2 + I_2$

put the value of I_3 in eq^n (1) and (2)

$50 = 15I_1 - 10I_2 - 5(2 + I_2)$

$\Rightarrow 50 = 15I_1 - 10I_2 - 10 - 5I_2$

$\Rightarrow 50 + 10 = 15I_1 - 15I_2$

$\Rightarrow 60 = 15I_1 - 15I_2$ —— (4)

$0 = 15I_1 - 12I_2 - 6(2 + I_2)$

$\Rightarrow 0 = 15I_1 - 12I_2 - 12 - 6I_2$

$\Rightarrow 0 + 12 = 15I_1 - 18I_2$

$\Rightarrow 12 = 15I_1 - 18I_2$ —— (5)

Substracting eq^n (5) from (4),

$\Rightarrow 48 = 3I_2$

$\Rightarrow I_2 = 16$ Amp

Putting the value of I_2 in eq^n (4),

$\Rightarrow 60 = 15I_1 - 15 \times 16$

$\Rightarrow 300 = 15I_1$

$\Rightarrow I_1 = 20$ Amp.

$$I_3 = 2 + I_2$$

$$= 2 + 16$$

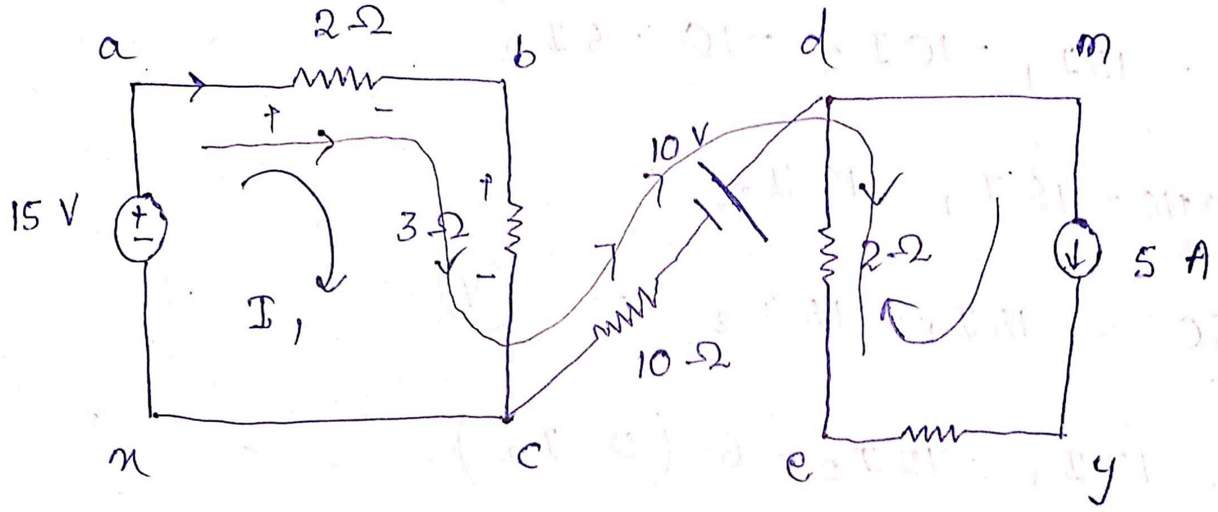
$$= 18 \text{ Amp}$$

$$V = I_3 \times 1$$

$$= 18 \times 1$$

$$= 18 \text{ volt} \quad (\text{Ans})$$

Q.



Find the voltage drop between the terminal a and e.

Sol^n

$$15 - 2I_1 - 3I_1 = 0$$

$$\Rightarrow 15 = 5I_1$$

$$\Rightarrow I_1 = 3 \text{ Amp}$$

$V_{ae} = V_a \pm V_{2\Omega} \pm V_{3\Omega} \pm V_{10\Omega} \pm 10V \pm V_{2\Omega} - V_e = 0$

$V_a \pm V_{2\Omega} \pm V_{3\Omega} \pm V_{10\Omega} \pm 10V \pm V_{2\Omega} - V_e = 0$

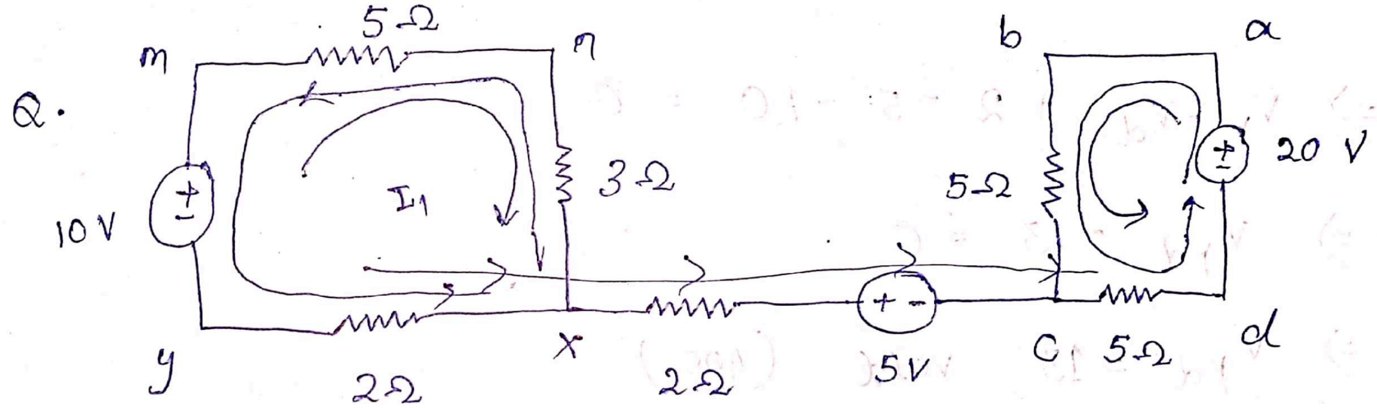
$V_a - 3\times 2 - 3\times 3 \pm 0 + 10 + 5\times 2 - V_e = 0$

$V_a - V_e = +6 + 9 - 10 - 10$

$V_{ae} = +15 - 20$

$V_{ae} = -5$ volt (Ans)

Q.



Find the voltage drop bet^n the terminal Y and d

Sol^n

$V_{Yd} = V_Y \pm V_{2\Omega} \pm V_{2\Omega} \pm 5V \pm V_{5\Omega} - V_d = 0$

For loop (1),

$+3I_1 + 5I_1 - 10 + 2I_1 = 0$

$\Rightarrow 10 I_1 = 10$

$\Rightarrow I_1 = 1$ Amp

for loop (2),

$+20 - 5I_2 - 5I_2 = 0$

$\Rightarrow 20 - 10I_2 = 0$

$\Rightarrow 20 = 10I_2$

$\Rightarrow I_2 = 2$ Amp.

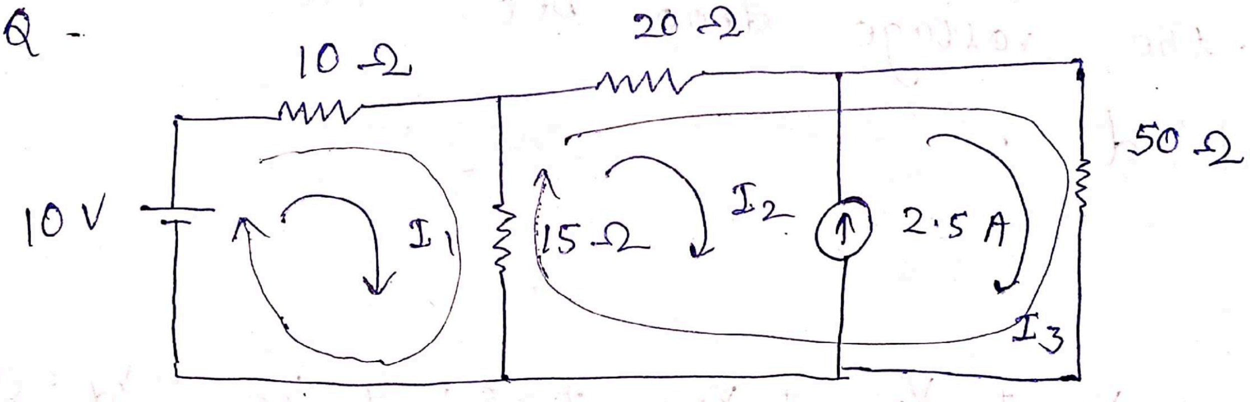
$V_Y + 2 \times 1 \pm 0 - 5 - 5 \times 2 - V_d = 0$

$\Rightarrow V_Y - V_d + 2 - 5 - 10 = 0$

$\Rightarrow V_{Yd} - 13 = 0$

$\Rightarrow V_{Yd} = 13$ volt (Ans.)

Q -



Find the current across 15 Ω resistor using mesh analysis.

for loop (1),

$$10 - 10I_1 - 15(I_1 - I_2) = 0$$

$$\Rightarrow 10 - 10I_1 - 15I_1 + 15I_2 = 0$$

$$\Rightarrow 10 = 25I_1 - 15I_2 \quad \text{—— (1)}$$

$$-20I_2 - 50I_3 - 15(I_2 - I_1) = 0$$

$$\Rightarrow -20I_2 - 50I_3 - 15I_2 + 15I_1 = 0$$

$$\Rightarrow 0 = 15I_1 - 35I_2 - 50I_3 \quad \text{—— (2)}$$

$$I_2 - I_3 = -2.5$$

$$\Rightarrow I_3 - I_2 = 2.5$$

$$\Rightarrow 2.5 = I_3 - I_2$$

$$\Rightarrow 2.5 = -I_2 + I_3 \quad \text{—— (3)}$$

$$\begin{bmatrix} 10 \\ 0 \\ 2.5 \end{bmatrix} = \begin{bmatrix} 25 & -15 & 0 \\ 15 & -35 & -50 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{bmatrix} 10 & -15 & 0 \\ 0 & -35 & -50 \\ 2.5 & -1 & 1 \end{bmatrix}}{\begin{bmatrix} 25 & -15 & 0 \\ 15 & -35 & -50 \\ 0 & -1 & 1 \end{bmatrix}}$$

$$= \frac{10(-35 - 50) + 15(0 + 125)}{25(-35 - 50) + 15(15 + 0)}$$

$$= \frac{10(-85) + 15(125)}{25(-85) + 225}$$

$$= \frac{-850 + 1875}{-2125 + 225}$$

$$= \frac{1025}{-1900}$$

$$= -0.539 \text{ Amp}$$

$$I_2 = \frac{\begin{bmatrix} 25 & 10 & 0 \\ 15 & 0 & -50 \\ 0 & 2.5 & 1 \end{bmatrix}}{\begin{bmatrix} 25 & -15 & 0 \\ 15 & -35 & -50 \\ 0 & -1 & 1 \end{bmatrix}}$$

$$= \frac{25(0 + 125) - 10(15 + 0)}{-1900}$$

$$= \frac{3125 - 150}{-1900}$$

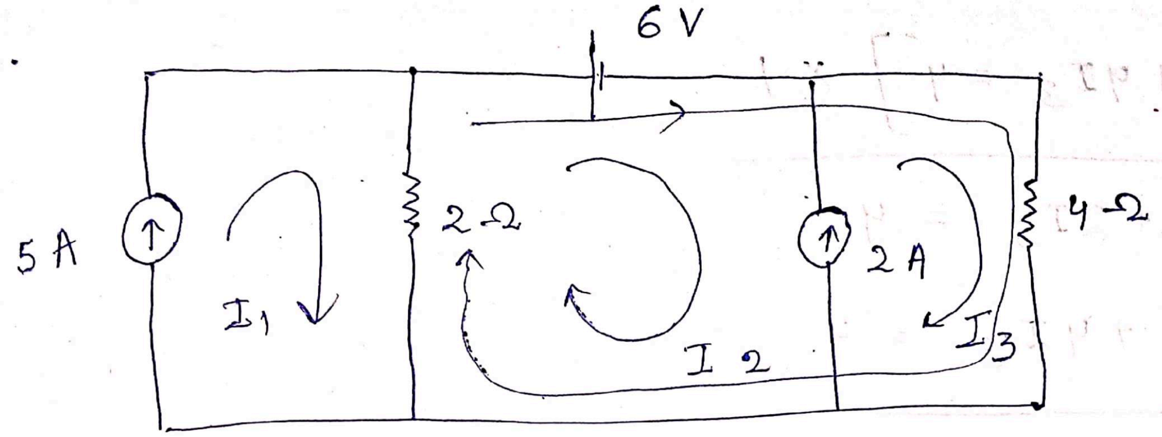
$$= \frac{2975}{-1900}$$

$$= -1.565 \text{ Amp}$$

$I_1 - I_2 = -0.539 + 1.565$

$= 1.026$ Amp.

Q.



Find the current across 4 Ω resistor, using mesh analysis.

Sol

$-6 - 4I_3 - 2(I_2 - I_1) = 0$

$\Rightarrow -6 - 4I_3 - 2I_2 + 2I_1 = 0$

$\Rightarrow 2I_1 - 2I_2 - 4I_3 - 6 = 0$ — (1)

$I_1 = 5$ A

$I_2 + 2 - I_3 = 0$

$2 = I_3 - I_2$ — (2)

Put $I_1 = 5$ A in eq^n (1),

$\Rightarrow 10 - 2I_2 - 4I_3 - 6 = 0$

$\Rightarrow -2I_2 - 4I_3 + 4 = 0$

$\Rightarrow -2I_2 - 4I_3 = -4$

$\Rightarrow 2I_2 + 4I_3 = 4$ — (3)

$[I_3 - I_2 = 2] \times 2$

$[2I_2 + 4I_3 = 4] \times 1$

$-2I_2 + 2I_3 = 4$

$2I_2 + 4I_3 = 4$

$6I_3 = 8$

$\Rightarrow I_3 = \frac{8}{6}$

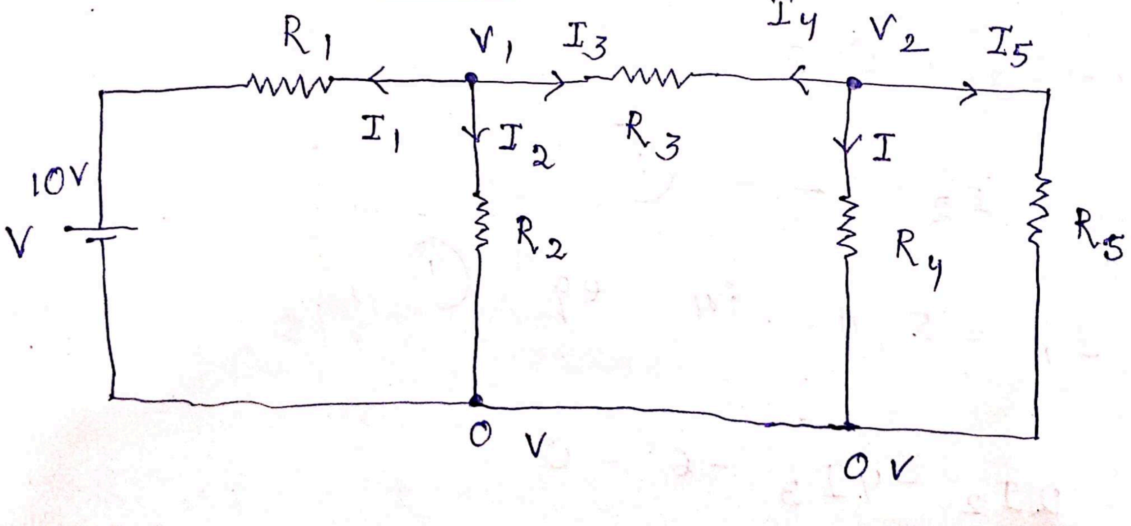
$= 1.333$ Amp (Ans)

$1.333 - I_2 = 2$

$\Rightarrow I_2 = 1.333 - 2$

$= -0.667$ (Amp)

Nodal Analysis :-



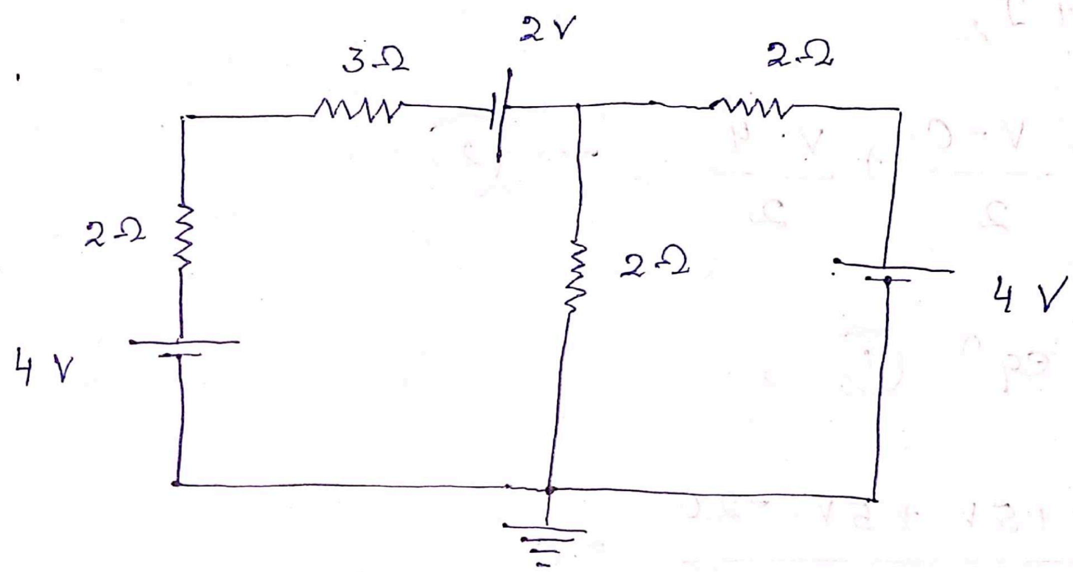
$I_1 + I_2 + I_3 = 0$

$$\Rightarrow \frac{V_1 - 10}{R_1} + \frac{V_1 - 0}{R_2} + \frac{V_1 - V_2}{R_3} = 0$$

$I_4 + I + I_5 = 0$

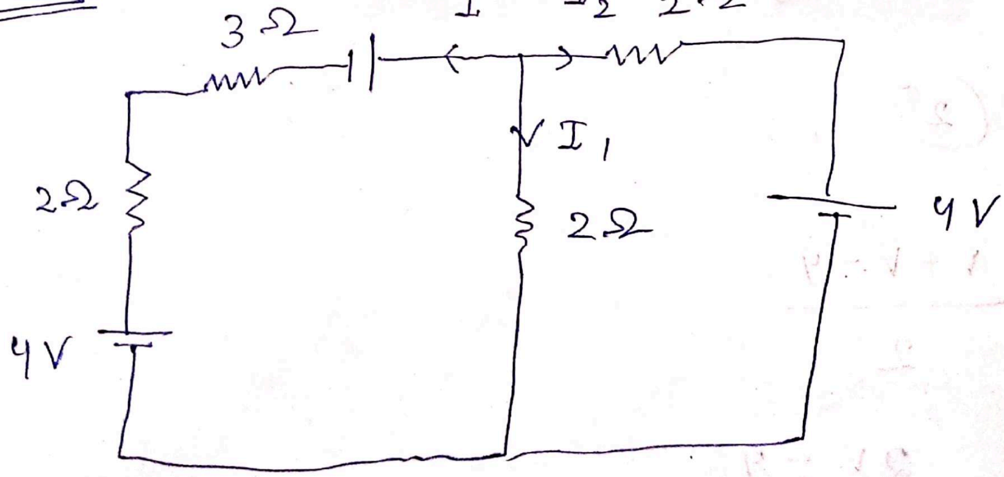
$$\Rightarrow \frac{V_2 - V_1}{R_3} + \frac{V_2 - 0}{R_4} + \frac{V_2 - 0}{R_5} = 0$$

Q.



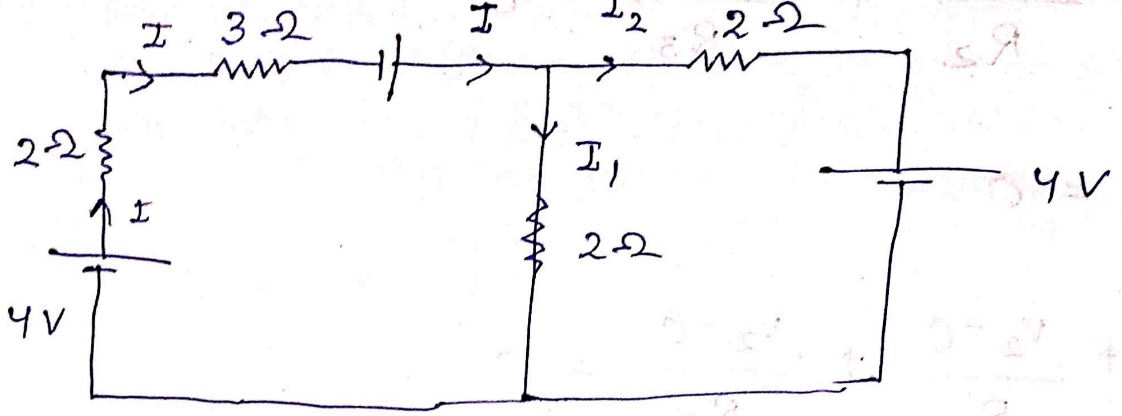
Using nodal analysis find the current across 2Ω resistor.

<u>sol</u>



$I + I_1 + I_2 = 0$

$$\frac{V-4}{2+3} + \frac{V-0}{2} + \frac{V-4}{2} = 0 \quad (1)$$



$$I = I_1 + I_2$$

$$\Rightarrow \frac{4-V}{2+3} = \frac{V-0}{2} + \frac{V-4}{2} \quad (2)$$

solving eq^n (1),

$$\frac{2V-8+5V+5V-20}{10} = 0$$

$$\Rightarrow 12V - 28 = 0$$

$$\Rightarrow V = \frac{28}{12} = \frac{7}{3} \, V$$

solving eq^n (2),

$$\frac{4-V}{5} = \frac{V+V-4}{2}$$

$$\Rightarrow \frac{4-V}{5} = \frac{2V-4}{2}$$

$\Rightarrow 8 - 2V = 10V - 20$

$\Rightarrow 12V = 28$

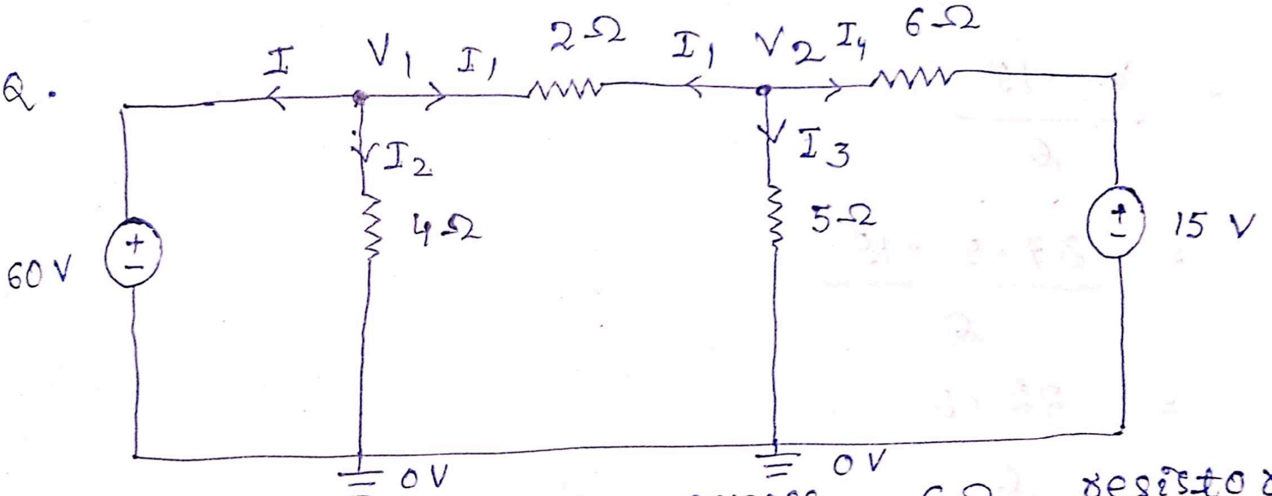
$\Rightarrow V = \frac{7}{3}$ volt

$$I_1 = \frac{V}{2}$$

$$= \frac{\frac{7}{3}}{2}$$

$$= \frac{7}{6} \text{ Amp}$$

Q.



Find the current across 6Ω resistor.

Solⁿ

By applying KCL,

at node (1),

$V_1 = 60$

at node (2),

$I_1 + I_3 + I_4 = 0$

$$\Rightarrow \frac{V_2 - V_1}{2} + \frac{V_2 - 0}{5} + \frac{V_2 - 15}{6} = 0$$

$$\Rightarrow \frac{15V_2 - 900 + 6V_2 + 5V_2 - 75}{30} = 0$$

$$\Rightarrow 26 V_2 - 975 = 0$$

$$\Rightarrow 26 V_2 = 975$$

$$\Rightarrow V_2 = \frac{975}{26}$$

$$= 37.5 \text{ V}$$

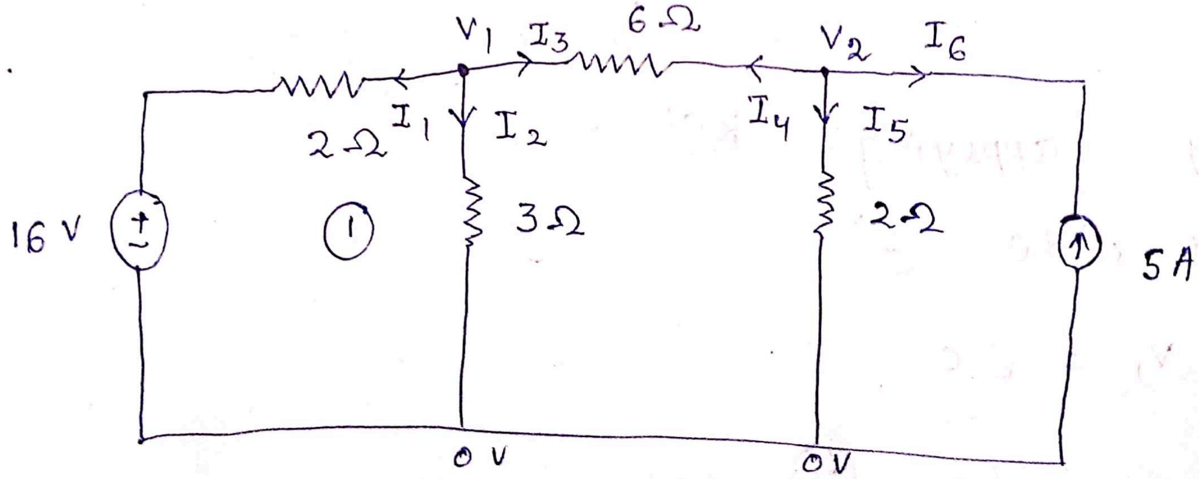
$$I_4 = \frac{V_2 - 15}{6}$$

$$= \frac{37.5 - 15}{6}$$

$$= \frac{22.5}{6}$$

$$= 3.75 \text{ Amp}$$

Q.



using nodal analysis find the current across 2Ω resistor.

Applying KCL, for node 1,

$I_1 + I_2 + I_3 = 0$

$\Rightarrow \frac{V_1 - 16}{2} + \frac{V_1 - 0}{3} + \frac{V_1 - V_2}{6} = 0$ — (1)

applying KCL at node 2,

$I_4 + I_5 + I_6 = 0$

$\Rightarrow \frac{V_2 - V_1}{6} + \frac{V_2 - 0}{2} + (-5) = 0$ — (2)

solving eqn (1).

$\Rightarrow \frac{3V_1 - 48 + 2V_1 + V_1 - V_2}{6} = 0$

$\Rightarrow 6V_1 - V_2 - 48 = 0$

$\Rightarrow 6V_1 - V_2 = 48$ — (3)

solving eqn (2),

$\Rightarrow \frac{V_2 - V_1 + 3V_2 - 30}{6} = 0$

$\Rightarrow 4V_2 - V_1 - 30 = 0$

$\Rightarrow -V_1 + 4V_2 = 30$ — (4)

eq^n (3) $\times 1 \Rightarrow 6V_1 - V_2 = 48$

eq^n (4) $\times 6 \Rightarrow -6V_1 + 24V_2 = 180$

$$23V_2 = 228$$

$$\Rightarrow V_2 = \frac{228}{23}$$

$$= 9.913 \text{ volt}$$

putting the value of V_2 in eq^n (3),

$6V_1 - V_2 = 48$

$\Rightarrow 6V_1 - 9.913 = 48$

$\Rightarrow 6V_1 = 57.913$

$\Rightarrow V_1 = 9.652$ volt

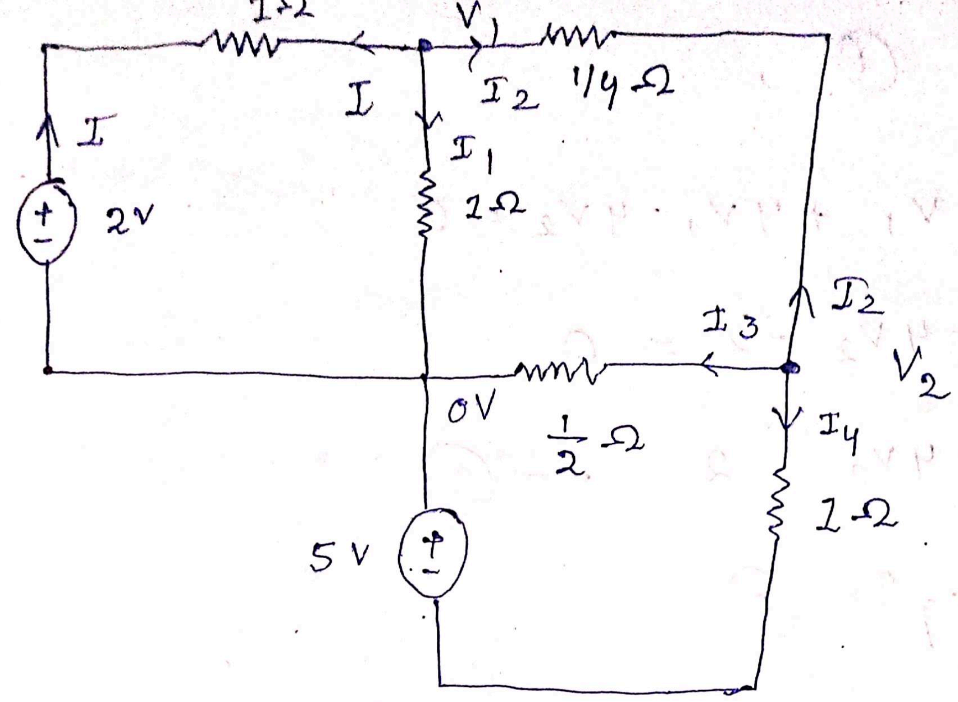
$$I_1 = \frac{V_1 - 16}{2}$$

$$= \frac{9.652 - 16}{2}$$

$= -3.174$ Amp

$= 3.174$ Amp

Q.



using nodal analysis find the current through 2V source.

Sol

Applying KCL at node - 1,

$$I + I_1 + I_2 = 0$$

$$\Rightarrow \frac{V_1 - 2}{1} + \frac{V_1 - 0}{1} + \frac{V_1 - V_2}{1/4} = 0 \quad \text{--- (1)}$$

Applying KCL at node - 2,

$$I_2 + I_3 + I_4 = 0$$

$$\Rightarrow \frac{V_2 - V_1}{1/4} + \frac{V_2 - 0}{1/2} + \frac{V_2 + 5}{1} = 0 \quad \text{--- (2)}$$

solving eq^n (1),

$\Rightarrow V_1 - 2 + V_1 + 4V_1 - 4V_2 = 0$

$\Rightarrow 6V_1 - 4V_2 - 2 = 0$

$\Rightarrow 6V_1 - 4V_2 = 2$ —— (3)

solving eq^n (2)

$4V_2 - 4V_1 + 2V_2 + V_2 + 5 = 0$

$\Rightarrow -4V_1 + 7V_2 = -5$ —— (4)

multiply $4 \times$ eq^n (3).

multiply $6 \times$ eq^n (4),

$\Rightarrow 24V_1 - 16V_2 = 8$

$\Rightarrow -24V_1 + 42V_2 = -30$

$26V_2 = -22$

$\Rightarrow V_2 = -0.846$ volt

$6V_1 - 4(-0.846) = 2$

$\Rightarrow 6V_1 + 3.384 = 2$

$\Rightarrow 6V_1 = 2 - 3.384$

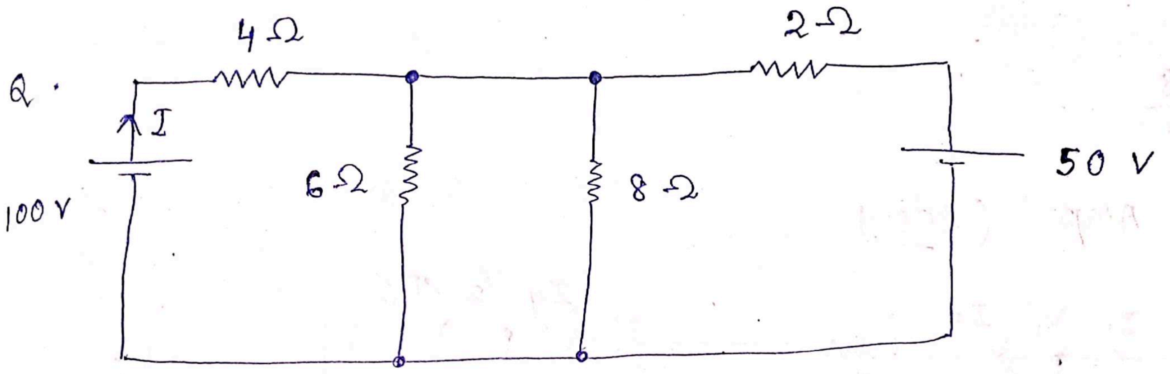
$\Rightarrow 6V_1 = -1.384$

$\Rightarrow V_1 = -0.23$ volt

$$I = \frac{V_1 - 2}{1}$$

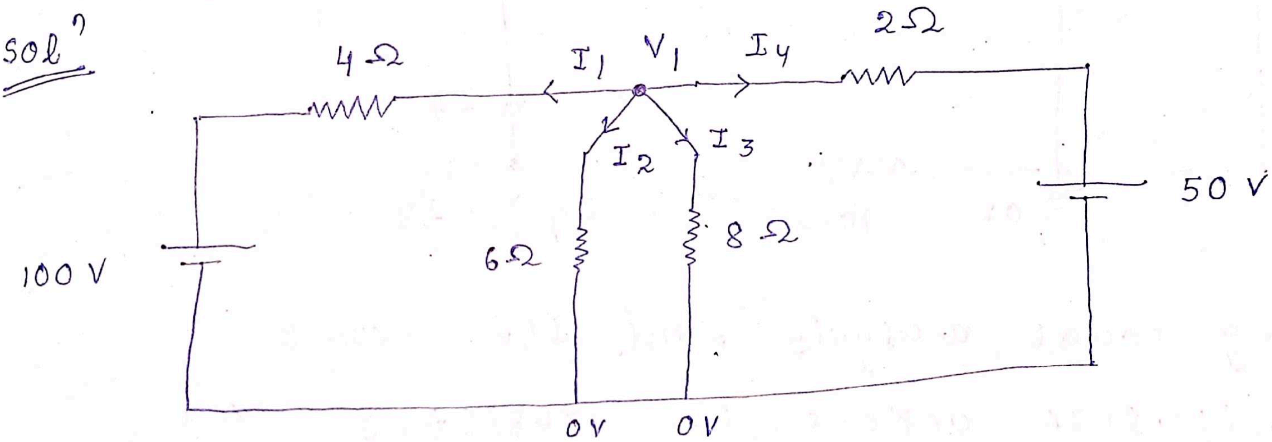
$$= \frac{-0.23 - 2}{1}$$

$$= -2.23 \text{ Amp}$$

Q. 

using nodal analysis find the current through 6Ω resistor.

Solⁿ



Appeying KCL at node 1,

$$I_1 + I_2 + I_3 + I_4 = 0$$

$$\Rightarrow \frac{V_1 - 100}{4} + \frac{V_1 - 0}{6} + \frac{V_1 - 0}{8} + \frac{V_1 - 50}{2} = 0$$

$$\Rightarrow \frac{6V_1 - 600 + 4V_1 + 3V_1 + 12V_1 - 600}{24} = 0$$

$$\Rightarrow 25V_1 = 1200$$

$\Rightarrow V_1 = \dfrac{1200}{25}$

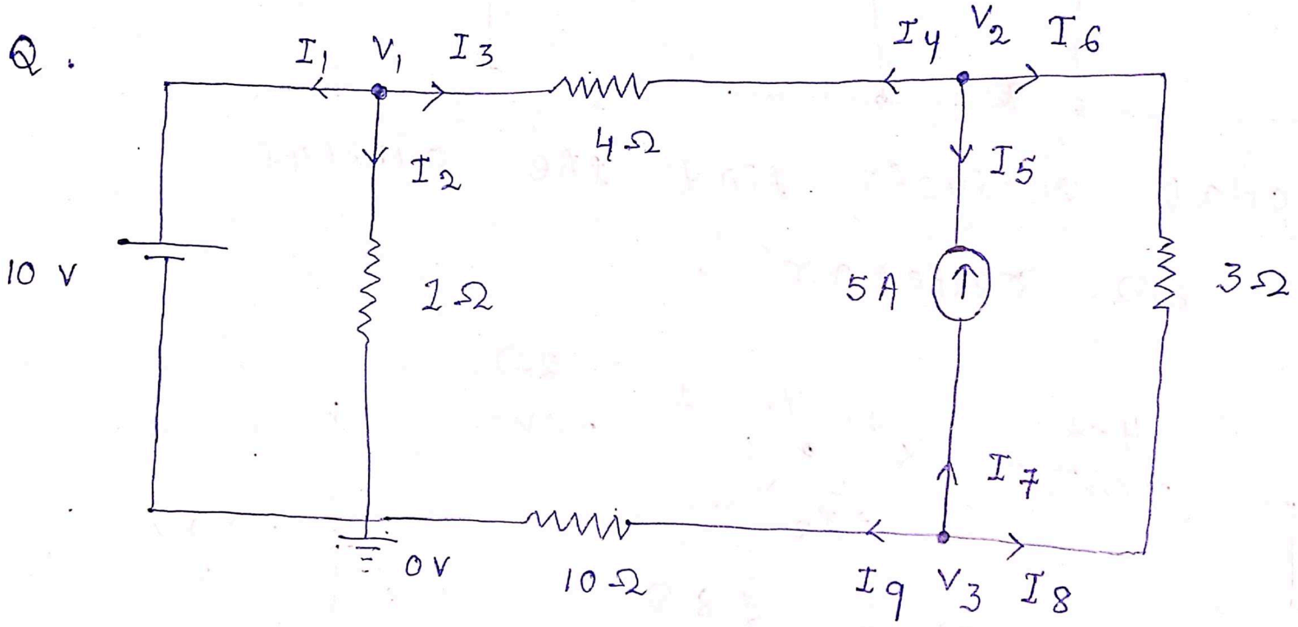
$= 48$ volt

$I_2 = \dfrac{V_1}{6}$

$= \dfrac{48}{6}$

$= 8$ Amp (Ans)

Q.



Using nodal analysis find the power dissipation across 4Ω resistor and current at 10Ω resistor.

Soln

~~Applying KCL at node 1,~~

$\cancel{I_1 + I_2 + I_3 = 0}$

$V_1 - 0 = 10$

$\Rightarrow V_1 = 10$ volt

Applying KCL at node 2,

$I_4 + I_5 + I_6 = 0$

$$\Rightarrow \frac{V_2 - V_1}{4} + (-5) + \frac{V_2 - \cancel{0}V_3}{3} = 0$$

$$\Rightarrow \frac{V_2 - V_1}{4} - 5 + \frac{V_2 - V_3}{3} = 0$$

$$\Rightarrow \cancel{\frac{3V_2 - 3V_1 - 60 + 4V_2}{12} = 0}$$

$$\Rightarrow \cancel{7V_2}$$

$$\Rightarrow \frac{3V_2 - 3V_1 - 60 + 4V_2 - 4V_3}{12} = 0$$

$$\Rightarrow -3V_1 + 7V_2 - 4V_3 = 60$$

$$\Rightarrow -30 + 7V_2 - 4V_3 = 60$$

$$\Rightarrow 7V_2 - 4V_3 = 90 \quad \text{--- } (1)$$

Applying KCL at node (3),

$I_7 + I_8 + I_9 = 0$

$\Rightarrow 5 + \frac{V_3 - V_2}{3} + \frac{V_3 - 0}{10} = 0$

$\Rightarrow \frac{150 + 10V_3 - 10V_2 + 3V_3}{30} = 0$

$\Rightarrow -10V_2 + 13V_3 = -150$ —— (2)

eq^n (1) $\times 10 \Rightarrow 70V_2 - 40V_3 = 900$

eq^n (2) $\times 7 \Rightarrow -70V_2 + 91V_3 = -1050$

$51V_3 = -150$

$\Rightarrow V_3 = \frac{-150}{51}$

$= -2.941$ volt

Putting the value of V_3 in eq^n (2),

$\Rightarrow -10V_2 + 13(-2.941) = -150$

$\Rightarrow -10V_2 - 38.233 = -150$

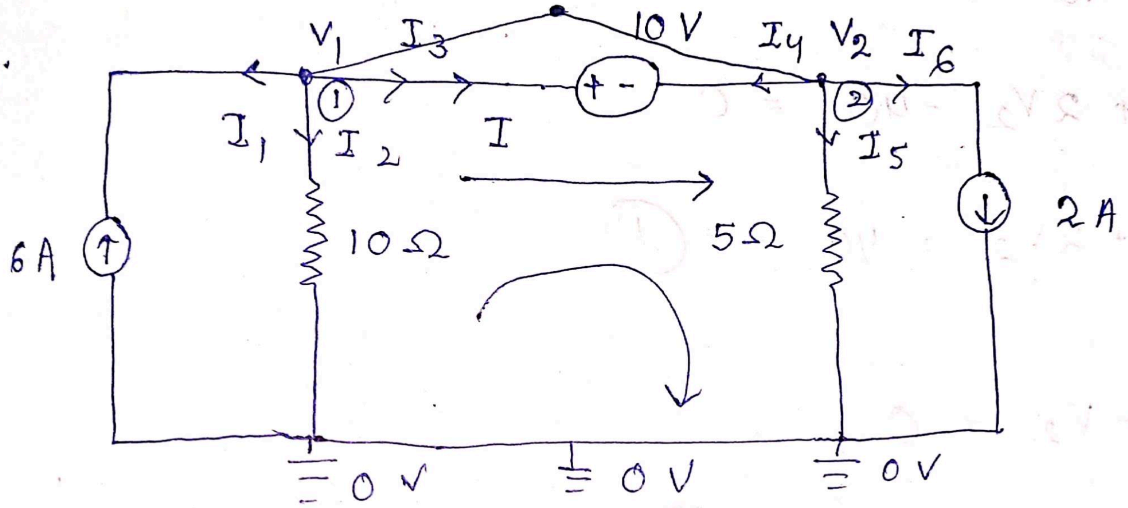
$\Rightarrow -10V_2 = -150 + 38.233$

$\Rightarrow -10V_2 = -111.767$

$\Rightarrow V_2 = 11.176$ volt

super node Analysis :~

Q.



using super node analysis find the current I.

soln

Applying KCL at node 1,

$$I_1 + I_2 + I_3 = 0$$

$$\Rightarrow -6 + \frac{V_1 - 0}{10} + I_3 = 0$$

Applying KCL at node 2,

$$I_4 + I_5 + I_6 = 0$$

$$\Rightarrow I_4 + \frac{V_2 - 0}{5} + 2 = 0$$

$\cancel{I_1 = I_2 + I_3}$

$I_1 + I_2 + I_5 + I_6 = 0$

$-6 + \frac{V_1}{10} + \frac{V_2}{5} + 2 = 0$

$\Rightarrow \frac{-60 + V_1 + 2V_2 + 20}{10} = 0$

$\Rightarrow V_1 + 2V_2 - 40 = 0$

$\Rightarrow V_1 + 2V_2 = 40$ — (1)

$V_1 - 10 - V_2 = 0.$

$\Rightarrow V_1 - V_2 = 10$ —— (2)

substracting eq^n (2) from eq^n (1),

$2V_2 + V_2 = 30$

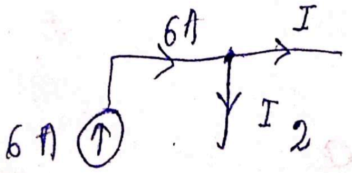
$\Rightarrow 3V_2 = 30$

$\Rightarrow V_2 = 10$ volt

putting the value of V_2 in eq^n (2),

$V_1 - 10 = 10$

$\Rightarrow V_1 = 20$ volt (Ans)



$6 = I_2 + I$

$\Rightarrow 6 = \frac{V_1}{10} + I$

$\Rightarrow 6 = \frac{20}{10} + I$

$\Rightarrow 6 = 2 + I$

$\Rightarrow I = 4$ Amp (Ans)

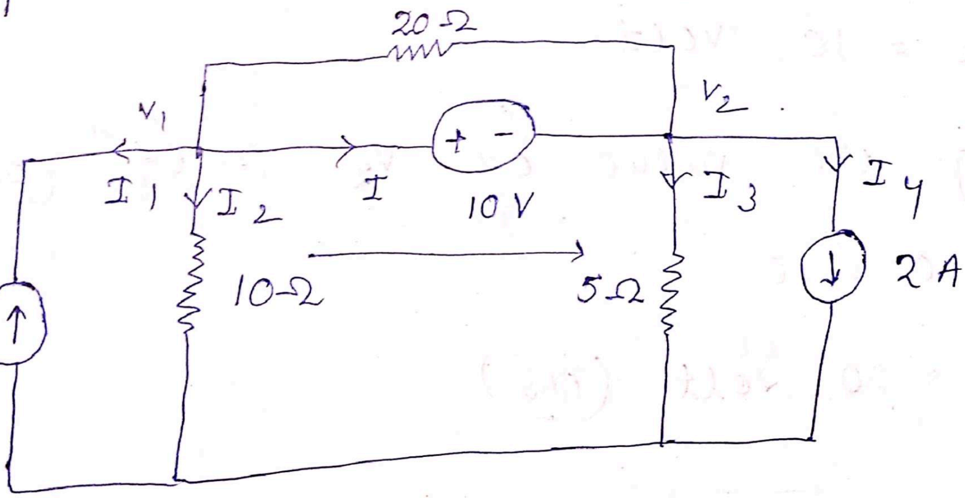
or

$I_4 + I_5 + I_6 = 0$

$\Rightarrow I_4 + 2 + 2 = 0$

$\Rightarrow I_4 = -4$ Amp (Ans) ($\because$ opposite direction)

Q.



Using super node analysis find the current I.

Sol^n $I_1 + I_2 + I_3 + I_4 = 0$

$$-6 + \frac{V_1}{10} + \frac{V_2}{5} + 2 = 0$$

$$\Rightarrow \frac{-60 + V_1 + 2V_2 + 20}{10} = 0$$

$$\Rightarrow V_1 + 2V_2 - 40 = 0$$

$$\Rightarrow V_1 + 2V_2 = 40 \quad \text{--- (1)}$$

$$V_1 - 10 - V_2 = 0$$

$$\Rightarrow V_1 - V_2 = 10 \quad \text{--- (2)}$$

Substracting eq^n (2) from eq^n (1),

$$2V_2 + V_2 = 30$$

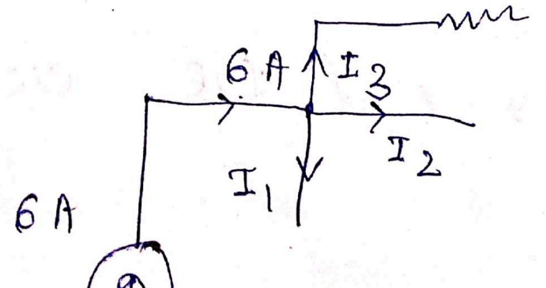
$$\Rightarrow 3V_2 = 30$$

$$\Rightarrow V_2 = 10 \text{ volt}$$

putting the value of V_2 in eq^n (2),

$$V_1 - 10 = 10$$

$$\Rightarrow V_1 = 20 \text{ volt} \quad \text{(Ans)}$$



(9)

$$6 = I_1 + I_2 + I_3$$

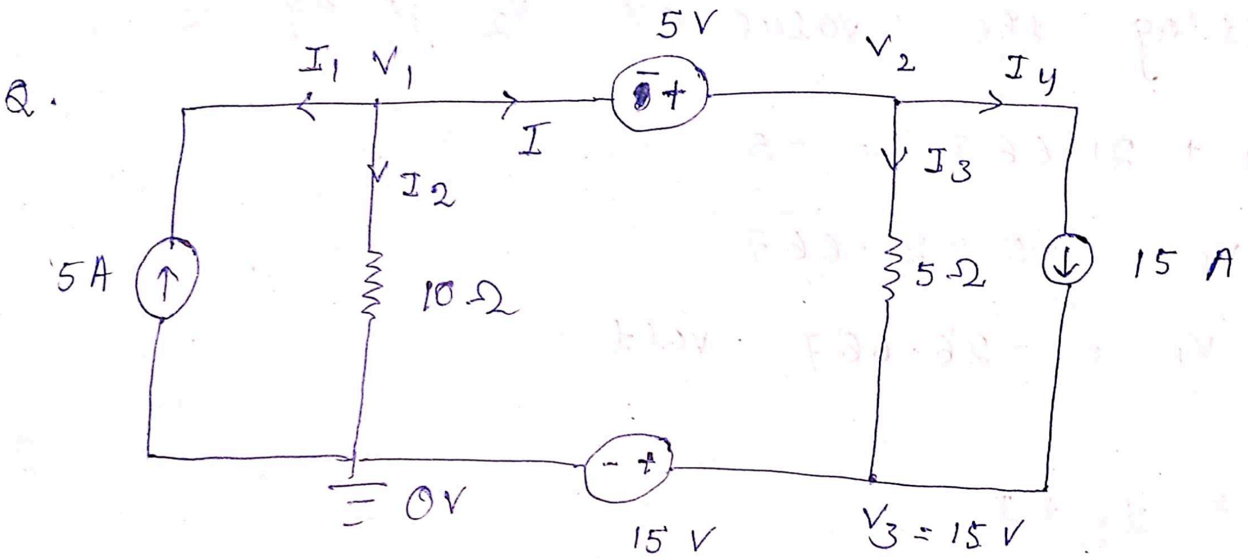
$$\Rightarrow 6 = \frac{V_1}{10} + I_2 + \frac{V_1 - V_2}{20}$$

$$\Rightarrow 6 = \frac{20}{10} + I_2 + \frac{20 - 10}{20}$$

$$\Rightarrow 6 = 2 + I_2 + 0.5$$

$$\Rightarrow I_2 = 3.5 \text{ Amp} \quad (\text{Ans})$$

Q.



using nodal analysis find the current 'I'.

Sol^n

$$I_1 + I_2 + I_3 + I_y = 0$$

$$\Rightarrow -5 + \frac{V_1}{10} + \frac{V_2 - V_3}{5} + 15 = 0$$

$$\Rightarrow -5 + \frac{V_1}{10} + \frac{V_2 - 15}{5} + 15 = 0$$

$$\Rightarrow \frac{V_1}{10} + \frac{V_2 - 15}{5} = -10$$

$\Rightarrow V_1 + 2V_2 - 30 = -100$

$\Rightarrow V_1 + 2V_2 = -70$ — (1)

$V_1 + 5 - V_2 = 0$

$\Rightarrow V_1 - V_2 = -5$ — (2)

subtracting eq^n (2) from eq^n (1),

$2V_2 + V_2 = -65$

$\Rightarrow 3V_2 = -65$

$\Rightarrow V_2 = -21.666$ volt

putting the value of V_2 in eq^n (2),

$V_1 + 21.667 = -5$

$\Rightarrow V_1 = -5 - 21.667$

$\Rightarrow V_1 = -26.667$ volt

$5 = I_2 + I$

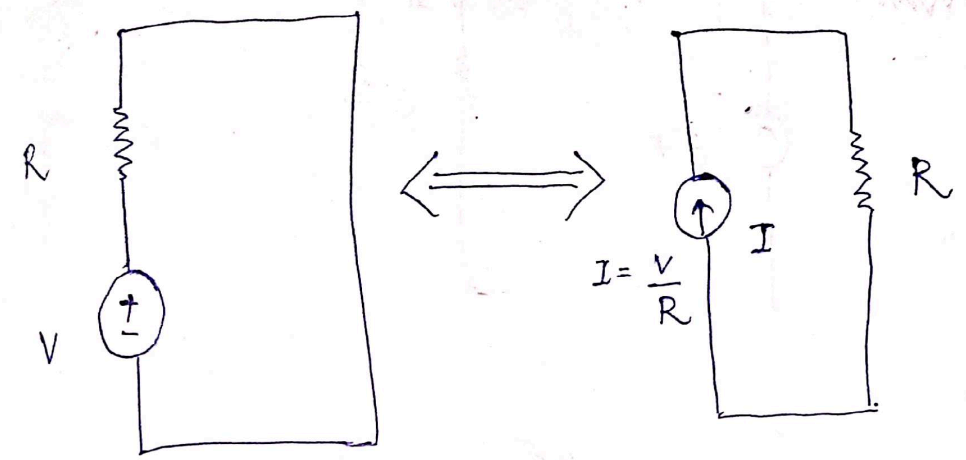
$\Rightarrow 5 = \frac{V_1}{10} + I$

$\Rightarrow 5 = \frac{-26.667}{10} + I$

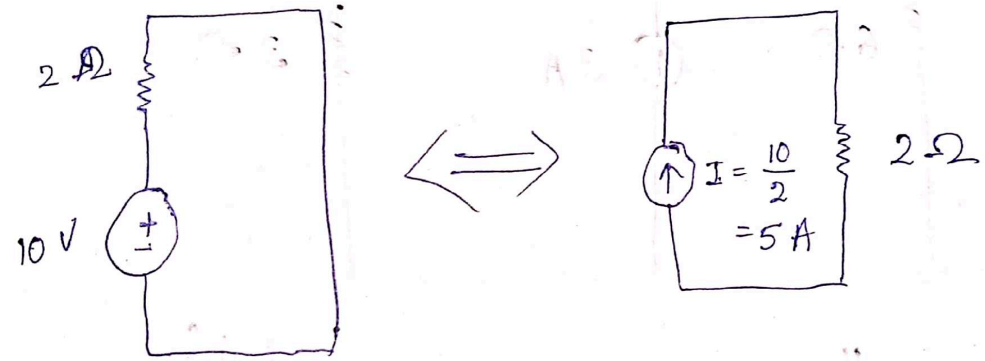
$\Rightarrow I = 5 + 2.6667$

$\Rightarrow I = 7.6667$ Amp (Ans)

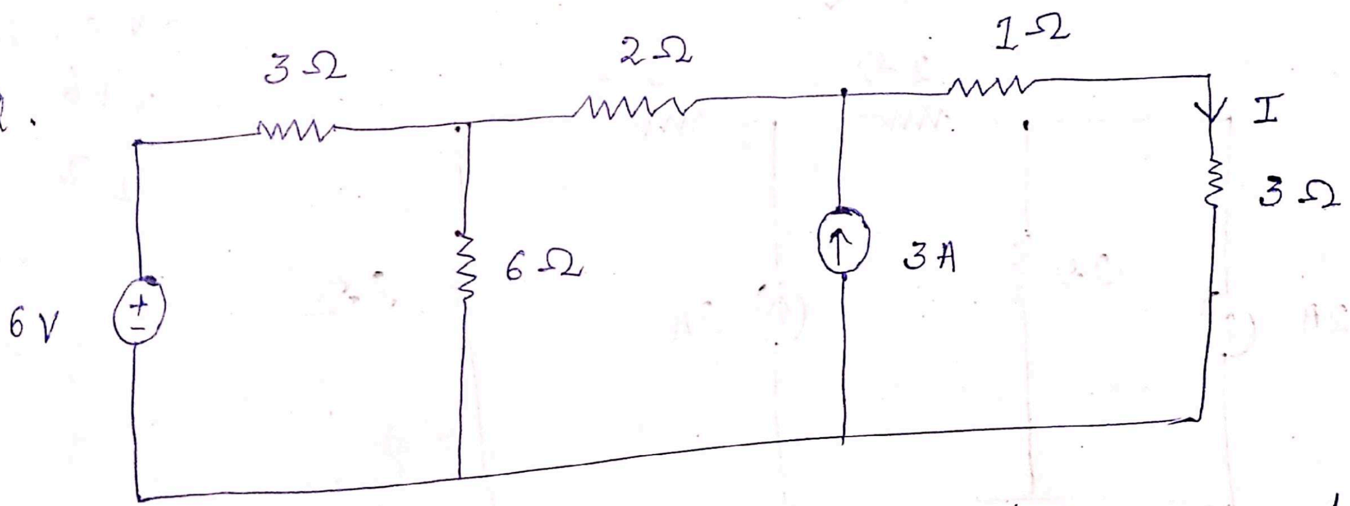
Source conversion Technique :-



Eg

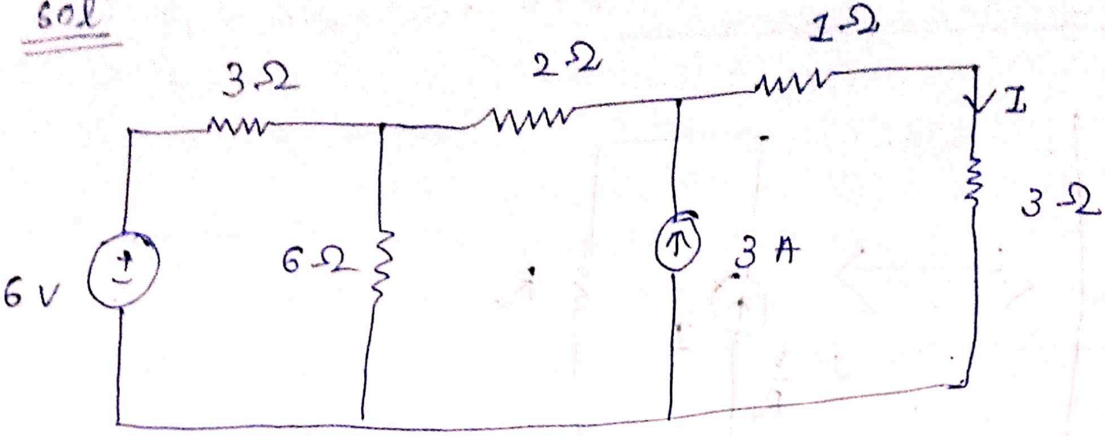


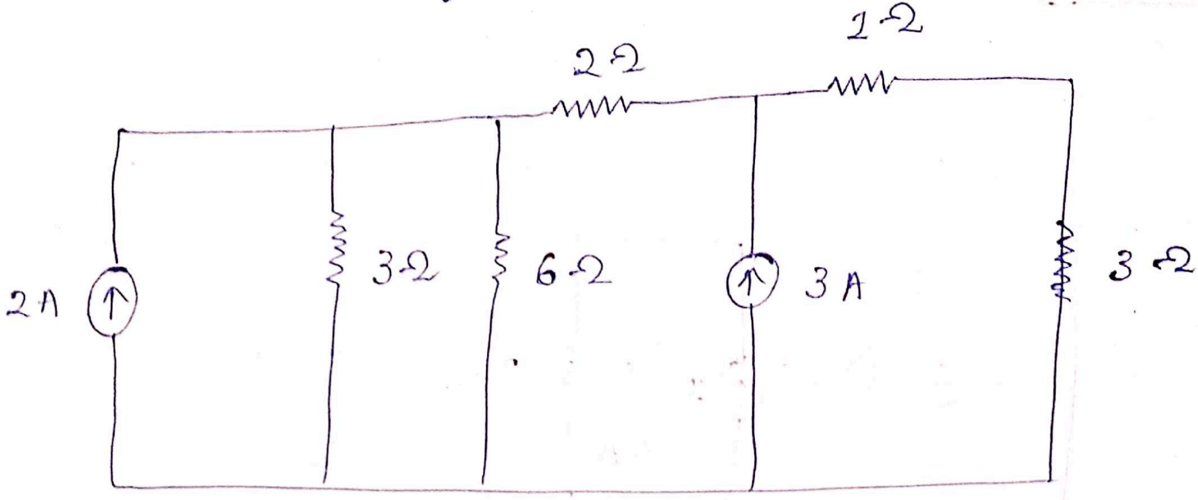
Q.



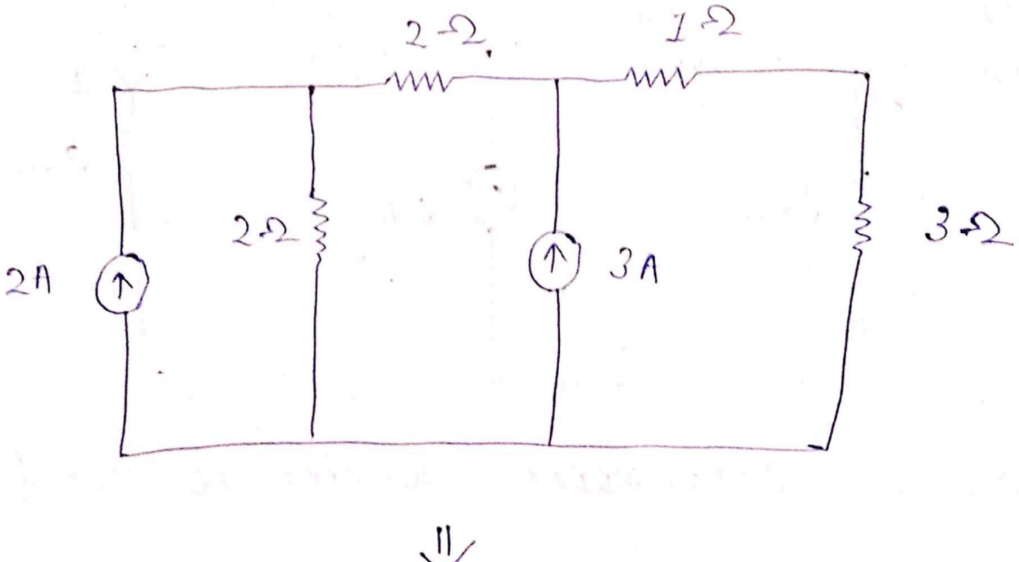
Using source conversion technique find the current I.

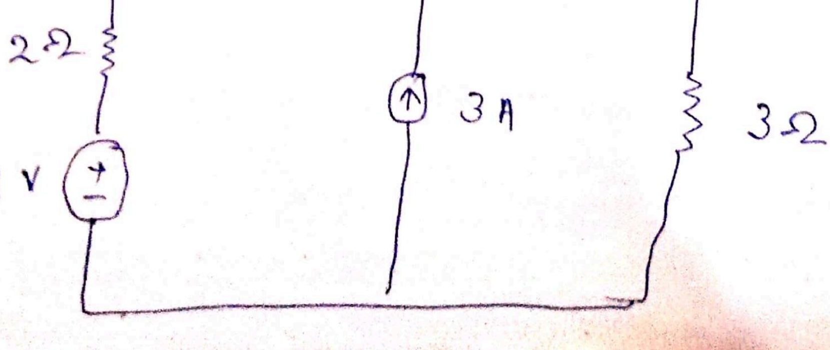
sol'

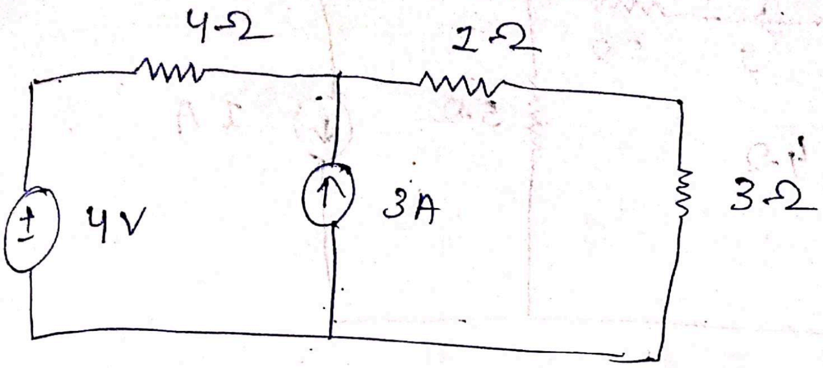


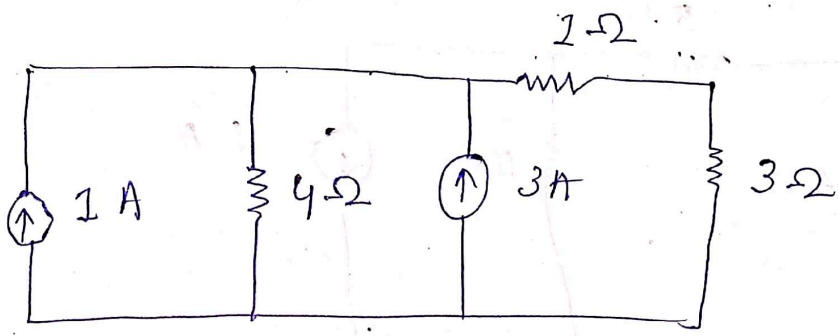


$3 \parallel 6 = \frac{3 \times 6}{3+6} = 2\,\Omega$

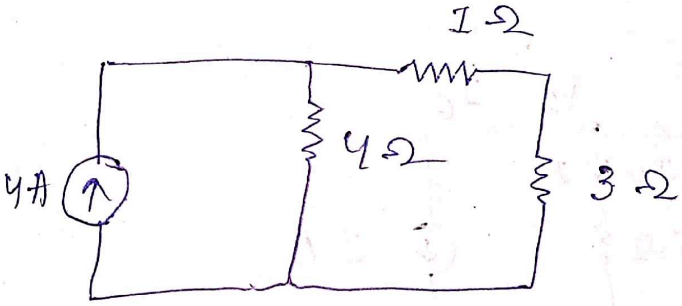


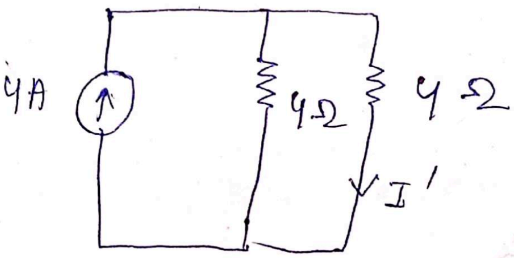






1 + 3 = 4A



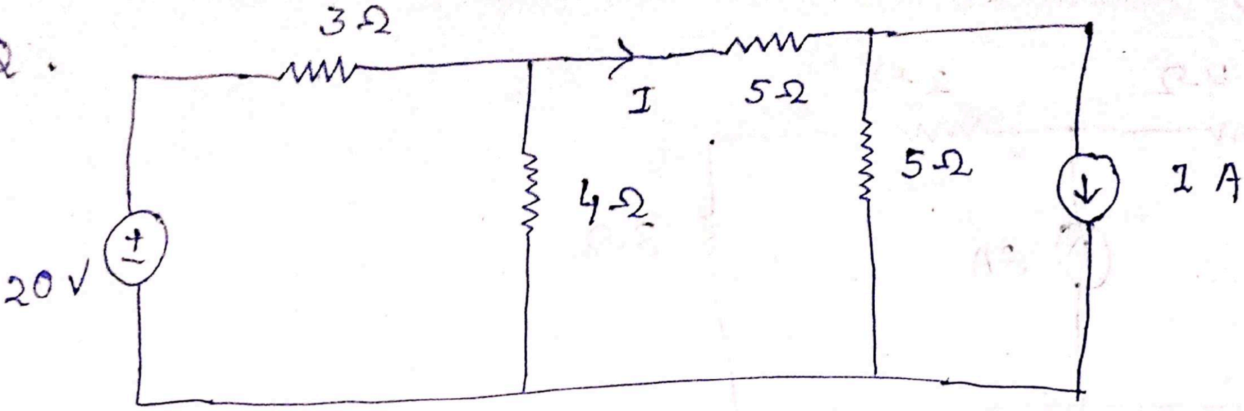


$$I' = \frac{I \times 4}{4+4}$$

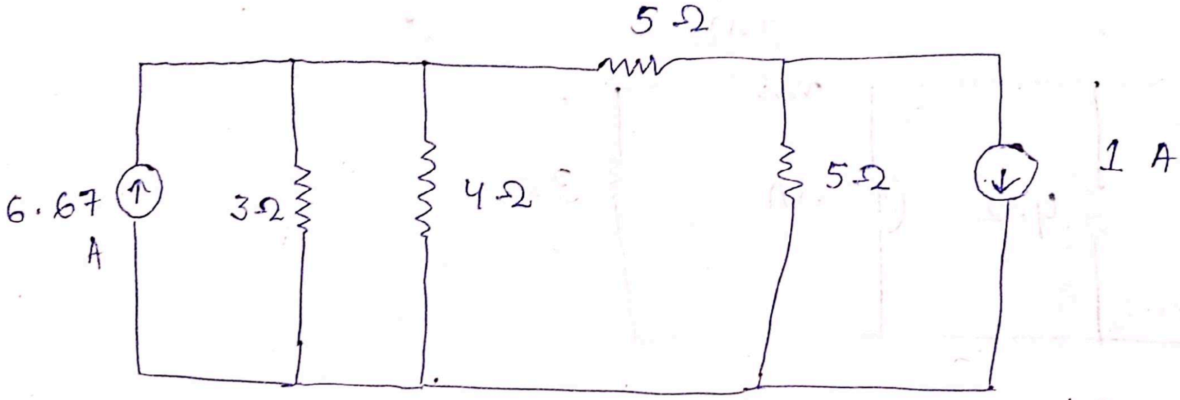
$$= \frac{4 \times 4}{4+4}$$

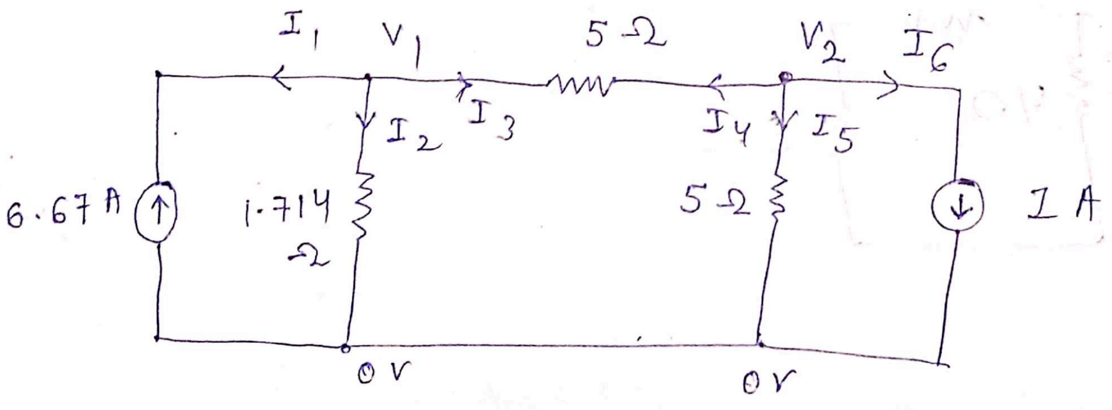
$= 2$ AMP

Q.



sol^n





For node ①,

$I_1 + I_2 + I_3 = 0$

$\Rightarrow \; -6.67 + \frac{V_1}{1.714} + \frac{V_1 - V_2}{5} = 0$

$\Rightarrow \; \frac{-57.161 + 5V_1 + 1.714V_1 - 1.714V_2}{8.57} = 0$

$\Rightarrow 6.714 V_1 - 1.714 V_2 = 57.161$ —— (1)

for node (2).

$I_4 + I_5 + I_6 = 0$

$\Rightarrow \frac{V_2 - V_1}{5} + \frac{V_2}{5} + 1 = 0$

$\Rightarrow \frac{V_2 - V_1 + V_2 + 5}{5} = 0$

$\Rightarrow -V_1 + 2V_2 = -5$

$\Rightarrow V_1 - 2V_2 = 5$ —— (2)

eqn (1) $\times 1 \Rightarrow 6.714 V_1 - 1.714 V_2 = 57.161$

eqn (2) $\times 6.714 \Rightarrow 6.714 V_1 - 13.428 V_2 = 33.57$

$\quad (-) \quad (+) \quad (-)$

$\Rightarrow 11.714 V_2 = 23.591$

$\Rightarrow V_2 = 2.013$ volt

$V_1 - 4.026 = 5$

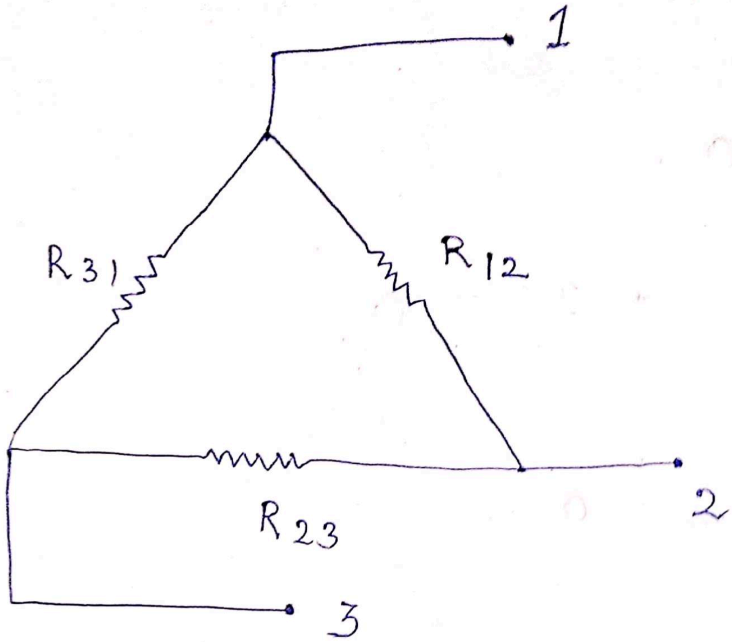
$\Rightarrow V_1 = 9.026$ volt (Ans)

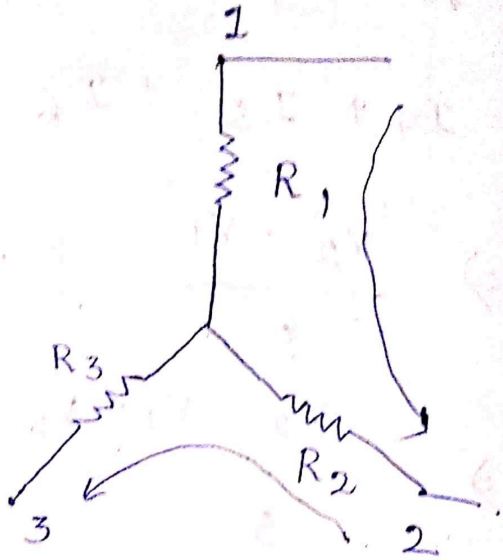
$I = \frac{V_1 - V_2}{5}$

$= \frac{9.026 - 2.013}{5}$

$= 1.402$ Amp (Ans)

Delta / star Transformation :-





$$R_{12} = \frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

$$R_{12} = R_1 + R_2$$

$$\frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} = R_1 + R_2 \quad \text{--- (1)}$$

$$R_{23} = R_{23} \parallel (R_{12} + R_{31})$$

$$R_{23} = R_2 + R_3$$

$$\Rightarrow R_{23} = \frac{R_{23} \times (R_{12} + R_{31})}{R_{23} + R_{12} + R_{31}}$$

$$R_{23} = R_2 + R_3 = \frac{R_{23} \times (R_{12} + R_{31})}{R_{12} + R_{23} + R_{31}} \quad \text{--- (2)}$$

$$R_{31} = R_{31} \parallel (R_{12} + R_{23}) \qquad R_{31} = R_3 + R_1$$

$$= \frac{R_{31} \times (R_{12} + R_{23})}{R_{31} + R_{12} + R_{23}}$$

$$R_{31} = \frac{R_{31} \times (R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} = R_3 + R_1 \quad \text{—— (3)}$$

subtracting eqⁿ (2) from eqⁿ (1),

$$R_1 + R_2 - (R_2 + R_3) = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} - \frac{R_{23}(R_{12} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

$$\Rightarrow R_1 - \cancel{R_2} - \cancel{R_2} - R_3 = \frac{\cancel{R_{12} R_{23}} + R_{12} R_{31} - \cancel{R_{23} R_{12}} - R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$\Rightarrow R_1 - R_3 = \frac{R_{12} R_{31} - R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} \quad \text{—— (4)}$$

~~Adding eqⁿ (3) and eqⁿ (4)~~ ,

$$R_1 + R_3 = \frac{R_{31} R_{12} + R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} \quad \text{—— (3)}$$

Adding eq^n (3) and eq^n (4),

$$R_1 + R_3 + R_1 - R_3 = \frac{R_{31} R_{12} + R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} + \frac{R_{12} R_{31} - R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$\Rightarrow 2R_1 = \frac{R_{31} R_{12} + \cancel{R_{31} R_{23}} + R_{12} R_{31} - \cancel{R_{23} R_{31}}}{R_{12} + R_{23} + R_{31}}$$

$$\Rightarrow 2R_1 = \frac{2 R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$\Rightarrow \boxed{R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}}$$

$$\boxed{R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}}$$

$$\boxed{R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}}$$

Star / delta Transformation :-

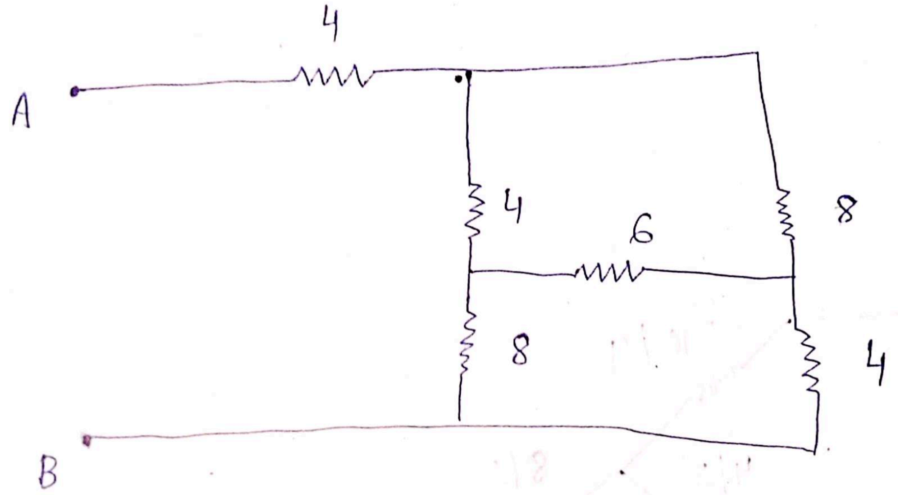
Multiplying eq^n (1) and (2), (2) and (3) and (3) and (1) and adding together and simplifying, we will get.

$$\boxed{R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}}$$

$$\boxed{R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}}$$

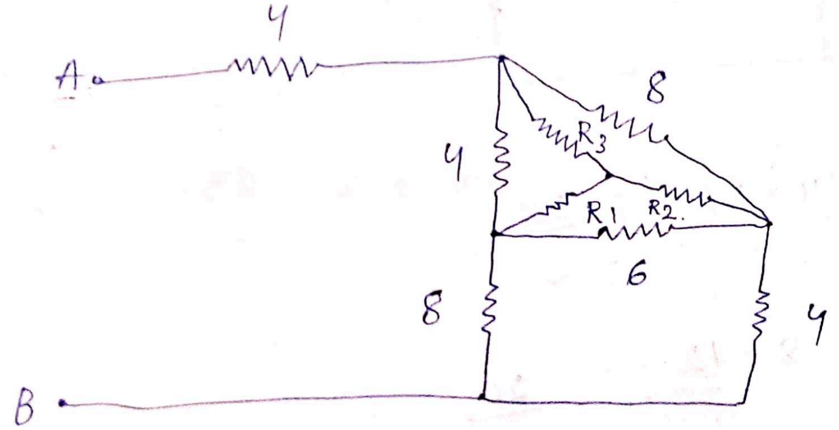
$$\boxed{R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}}$$

Q.



find R_{AB} ?

sol^n



$$R_1 = \frac{4 \times 6}{4 + 6 + 8} = \frac{24}{18} = \frac{4}{3}$$

$$R_2 = \frac{8 \times 6}{4+6+8}$$

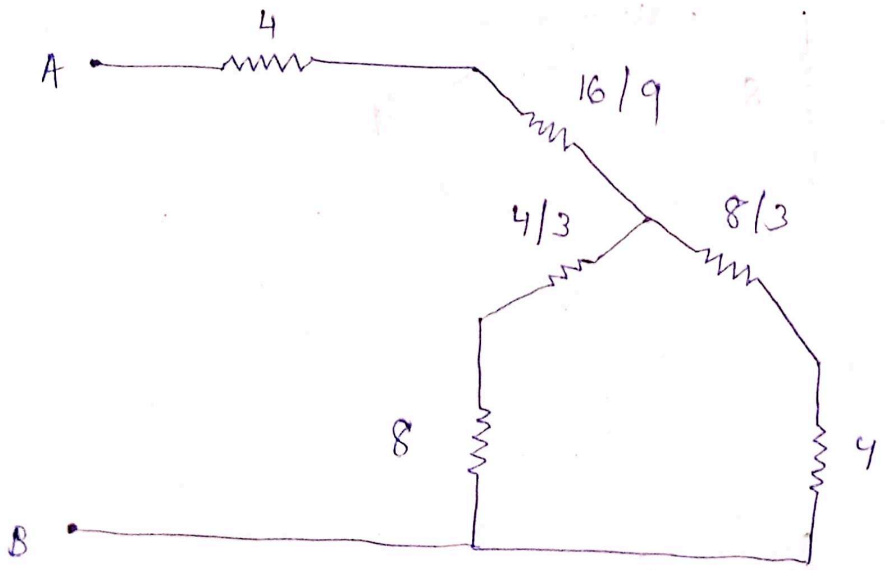
$$= \frac{48}{18}$$

$$= \frac{8}{3}$$

$$R_3 = \frac{4 \times 8}{4+6+8}$$

$$= \frac{32}{18}$$

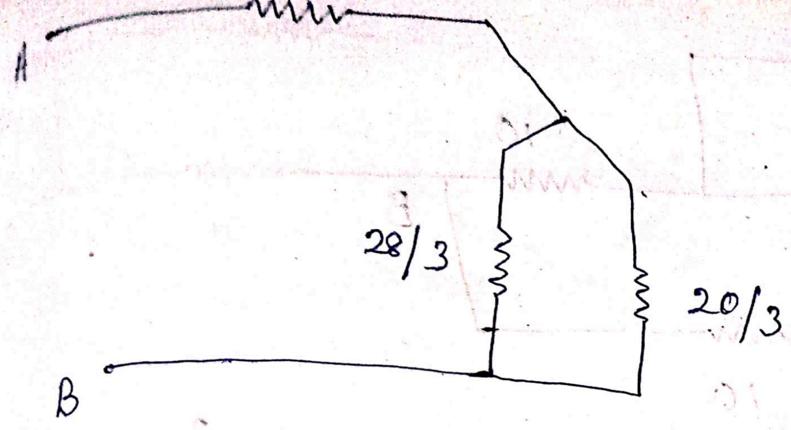
$$= \frac{16}{9}$$



$$\frac{4}{3} + 8 = \frac{4+24}{3} = \frac{28}{3}$$

$$\frac{8}{3} + 4 = \frac{8+12}{3} = \frac{20}{3}$$

$$4 + \frac{16}{9} = \frac{36+16}{9} = \frac{52}{9}$$

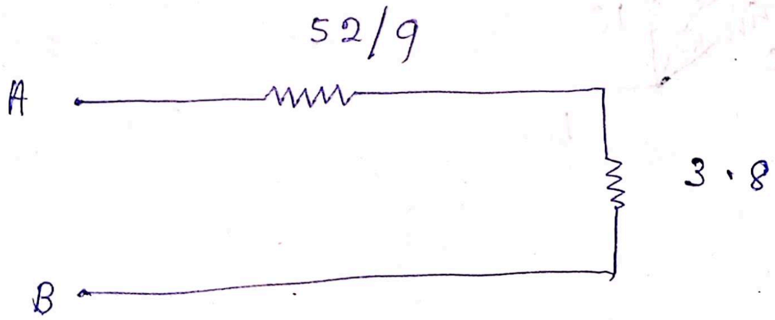


$$\frac{28}{3} \parallel \frac{20}{3} = \frac{\frac{28}{3} \times \frac{20}{3}}{\frac{28}{3} + \frac{20}{3}}$$

$$= \frac{\frac{560}{9}}{\frac{48}{3}}$$

$$= \frac{560}{3 \times 48}$$

$$= 3.8$$

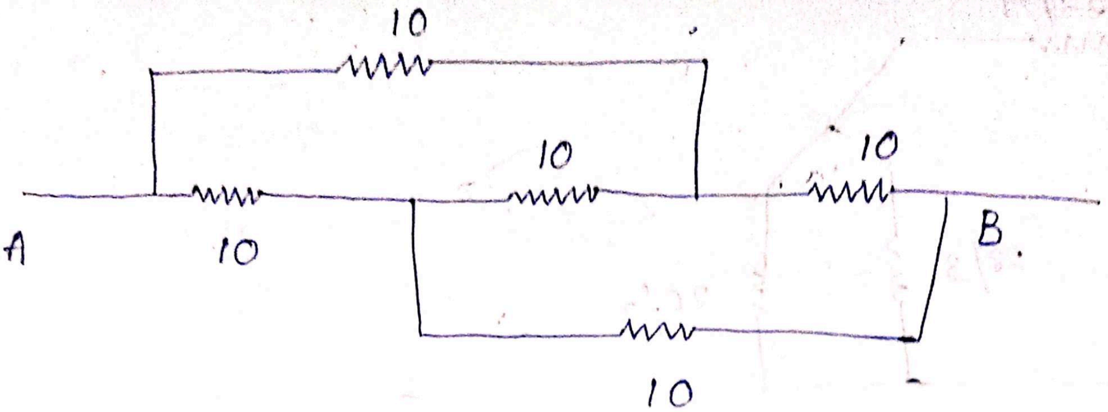


$$R_{AB} = \frac{52}{9} + 3.8$$

$$= 5.7 + 3.8$$

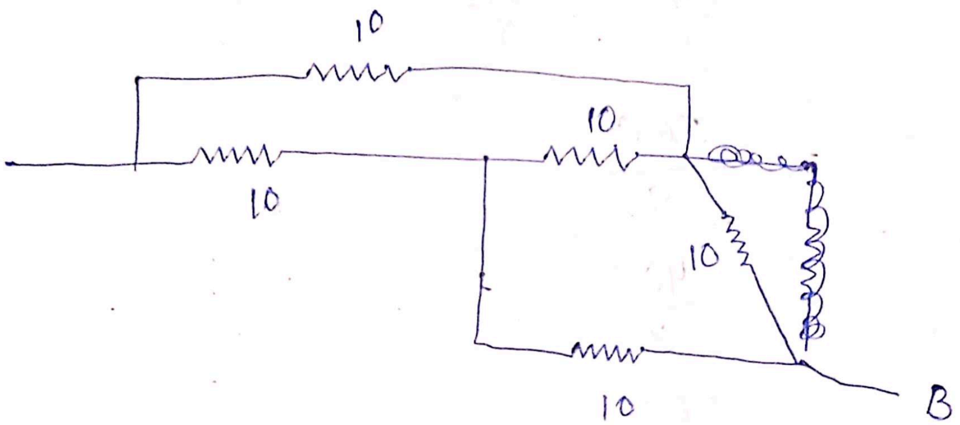
$$= 9.5\ \Omega.$$

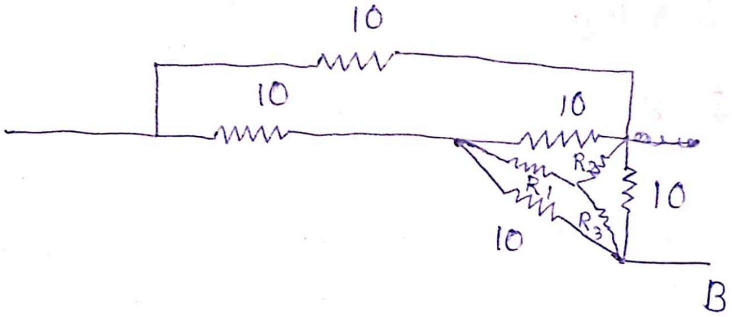
Q.



Find the equivalent resistance bet^n the terminal A and B.

Sol^n



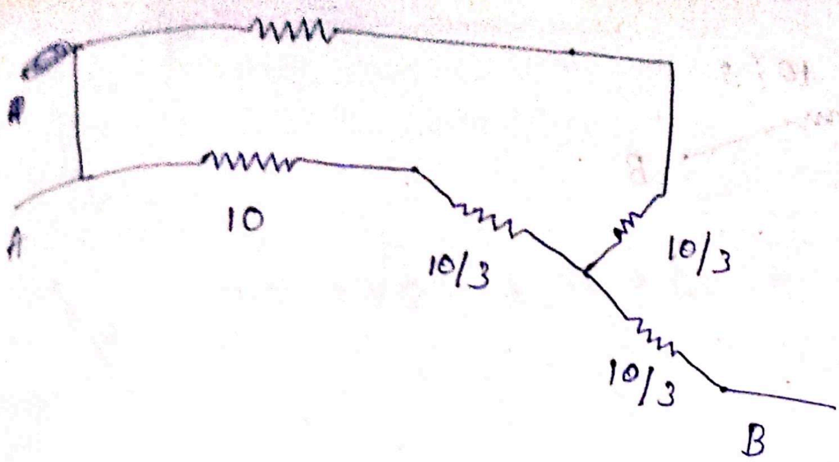


$$R_1 = \frac{10 \times 10}{10 + 10 + 10}$$

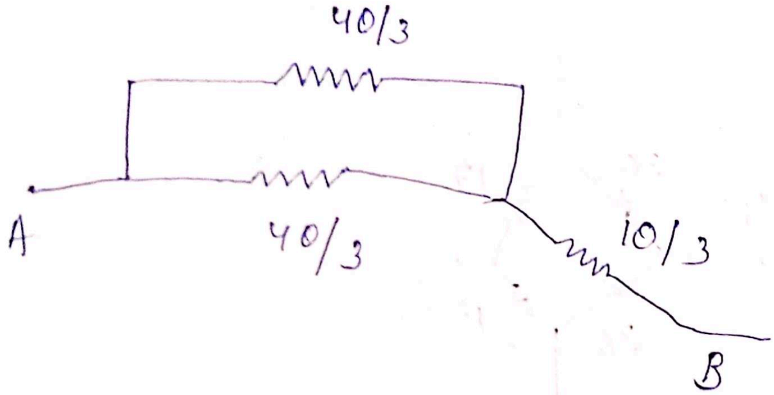
$$= \frac{100}{30}$$

$$= \frac{10}{3}$$

$$R_1 = R_2 = R_3 = \frac{10}{3}$$



$$10 + \frac{10}{3} = \frac{30+10}{3} = \frac{40}{3}$$



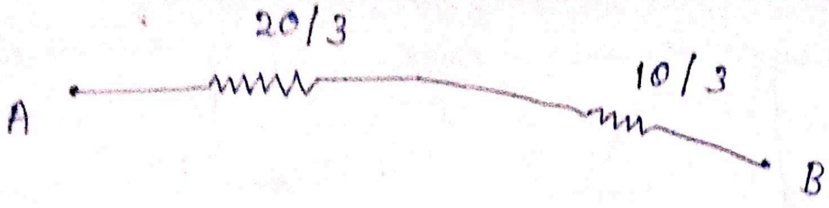
$$\frac{40}{3} \parallel \frac{40}{3}$$

$$= \frac{\frac{40}{3} \times \frac{40}{3}}{\frac{40}{3} + \frac{40}{3}}$$

$$= \frac{\frac{1600}{9}}{\frac{80}{3}}$$

$$= \frac{1600}{9} \times \frac{3}{80}$$

$$= \frac{20}{3}$$

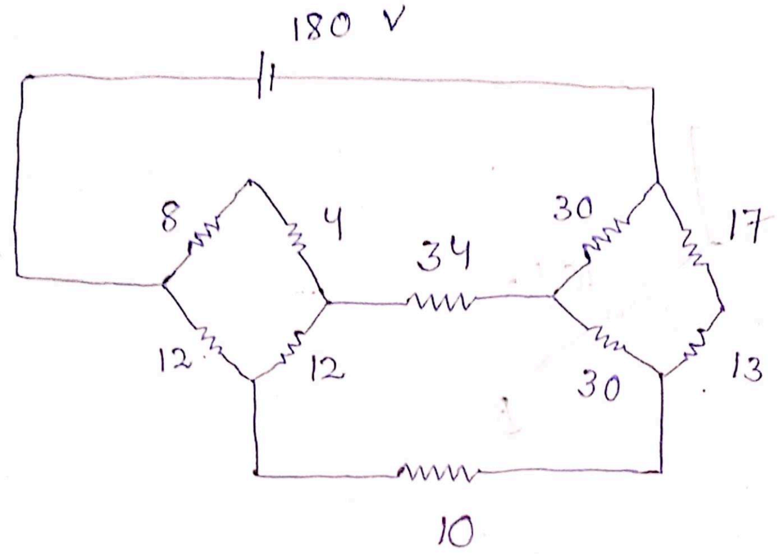


$$R_{AB} = \frac{20}{3} + \frac{10}{3}$$

$$= \frac{30}{3}$$

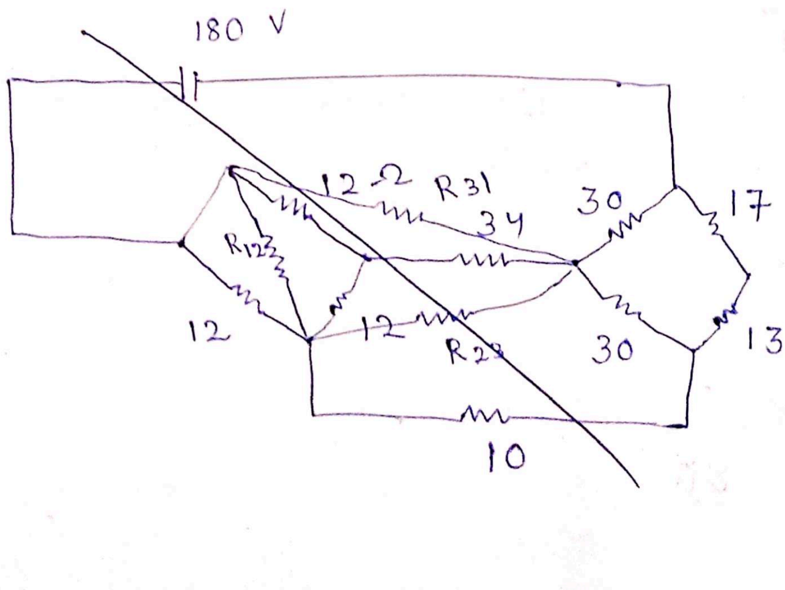
$$= 10\ \Omega$$ (Ans)

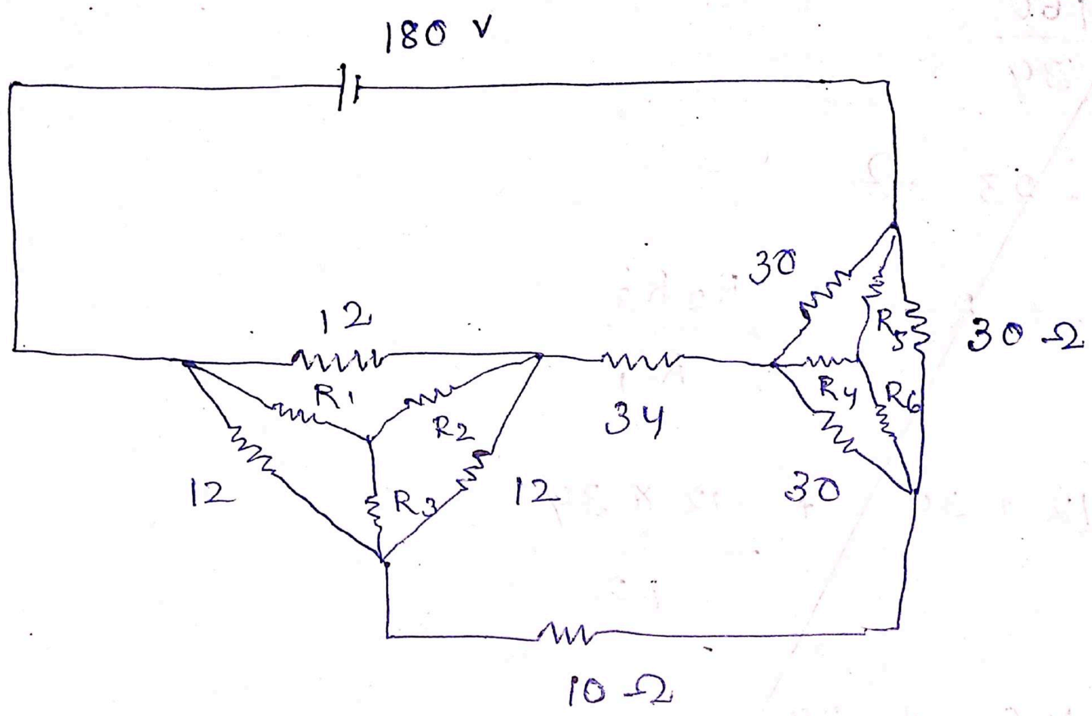
Q.



calculate the current through $10\ \Omega$ resistor.

Sol^n





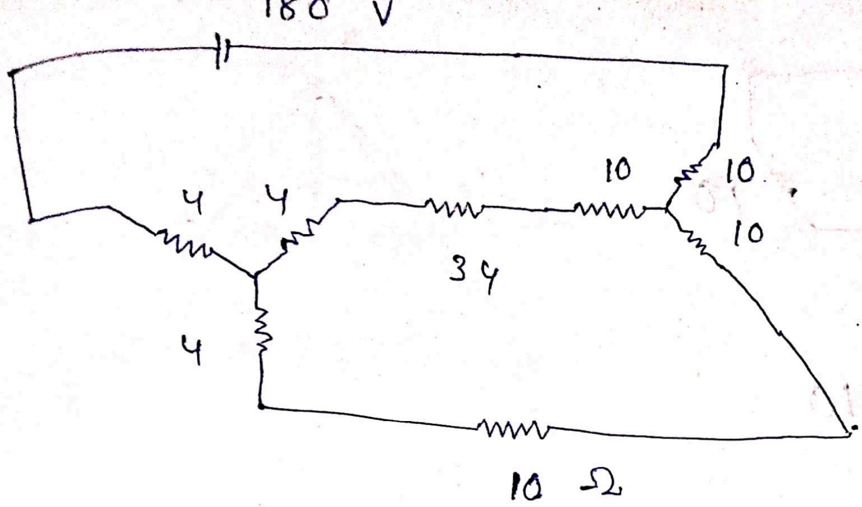
$$R_1 = \frac{12 \times 12}{12 + 12 + 12}$$

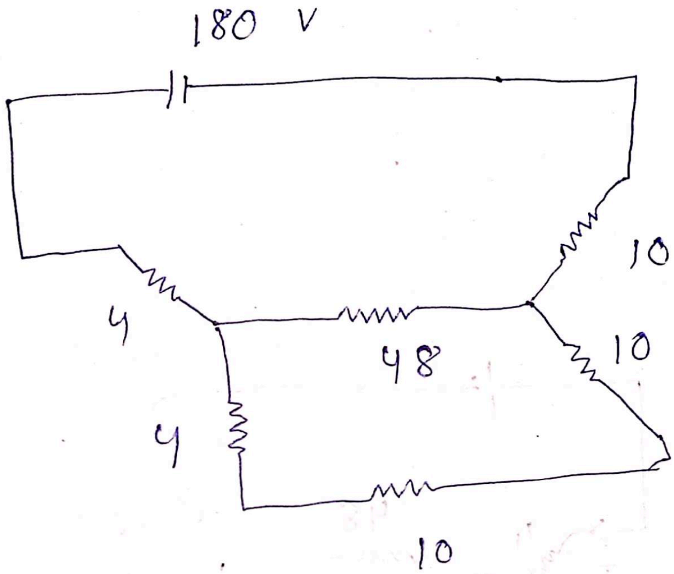
$$= \frac{144}{36}$$

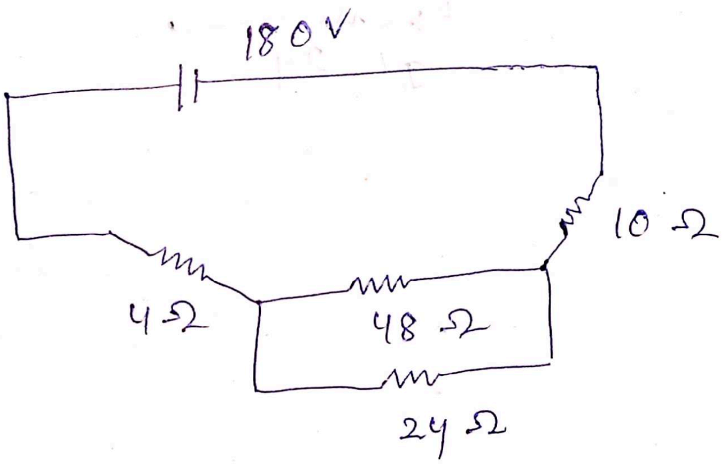
$$= 4\,\Omega = R_2 = R_3$$

$$R_4 = \frac{30 \times 30}{30 + 30 + 30}$$

$$= \frac{900}{90} = 10\,\Omega = R_5 = R_6$$







$$48 \parallel 24 = \frac{48 \times 24}{48 + 24}$$

$$= \frac{1152}{72}$$

$$= 16$$